DEFINITION OF A THREE-DIMENSIONAL SPATIAL DATA MODEL FOR SOUTHEASTERN WISCONSIN

SOUTHEASTERN WISCONSIN REGIONAL PLANNING COMMISSION

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Mr. Philip C. Evenson, Executive Director
Southeastern Wisconsin Regional Planning Commission
P.O. Box 1607
Waukesha, Wisconsin 53187

Dear Mr. Evenson,

Transmitted herewith is the report entitled, "Definition of a Three-Dimensional Spatial Data Model for Southeastern Wisconsin." The report describes an arrangement of existing concepts, procedures, and equations identified as a Global Spatial Data Model (GSDM) and is intended to support continued expansion and use into the future of the horizontal and vertical survey control networks established within southeast Wisconsin by the Commission. Through application of the model:

- Existing horizontal and vertical spatial data collected and used by the Commission for more than 30 years can be combined into a single three-dimensional digital spatial data base.

- Spatial data collected by modern measurement systems such as global positioning system (GPS) surveying and digital photogrammetric mapping procedures can be efficiently combined with traditional surveying and mapping data with no loss of geometrical integrity.

- The decentralized use of digital spatial data for such activities as land surveying, engineering surveying, and the creation of planning and engineering data bases is facilitated.

Although the described procedures and the rules of solid geometry for manipulating spatial data are equally applicable the world over, the GSDM relies on the use of local coordinate differences which means the local perspective (plane surveying in three dimensions) is preserved at all times. Therefore, the GSDM is specifically and immediately appropriate for local adoption and use, especially in the seven-county southeastern Wisconsin planning area which has an excellent three-dimensional survey control network already in place.

Thank you for the opportunity to be of service to the Commission.

Yours truly

Earl F. Burkholder

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Consulting Geodetic Engineer

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DEFINITION OF A THREE-DIMENSIONAL (3-D) SPATIAL DATA MODEL FOR SOUTHEASTERN WISCONSIN

ABSTRACT

The goal of this study was to examine and redefine efficient procedures for collection, storage, manipulation and use of spatial data in a three-dimensional (3-D) environment. Impetus for the study includes the need to make better use of existing resources for managing spatial information in an environment dictated by use of the modern digital computer and digital data bases. The overriding criterion was to identify a simple model which would support a local perspective in a global framework. The result of the study is a description of a universal spatial data model which exploits the tools of modern technology, preserves the proud heritage of past surveying practices, and accommodates a local view of the world while preserving true geometrical spatial relationships on a global scale. The model optionally permits assignment of standard deviations to uncorrelated quantities and accommodates input of a full covariance matrix for the coordinates of a point in either a global or local reference frame. As a consequence, the spatial quality (3-D positional tolerance) of each point so defined is always readily available and provides an efficient mechanism by which spatial data (or derived results) can be judged acceptable for a given application.

INTRODUCTION

The Southeastern Wisconsin Regional Planning Commission (SEWRPC) has, over the past 30 years, provided leadership both locally and nationally in developing parcel-based Land Information Systems (Bauer, 1994). The SEWRPC spatial data base utilizes the North American Datum of 1927 (NAD27) for horizontal and the National Geodetic Vertical Datum of 1929 (NGVD29) for vertical referencing. Extensive field surveys have been conducted over the years to establish accurate survey monuments throughout the seven-county Region.

Although spatial data users in the SEWRPC Region are well-served by the accuracy, integrity and sufficiency of the existing networks, the Commission has come under pressure to adopt the North American Datum of 1983-1991 Adjustment (NAD83(91) for horizontal referencing and the North American Vertical Datum of 1988 (NAVD88) for vertical referencing. The Commission adamantly defends continued use of NAD27 and NGVD29, but recognizes the need for proven relationships between the various datums throughout the Region. To preserve and enhance the value of existing spatial data, the Commission prepared two separate technical reports:

> SEWRPC Technical Report No. 34, "A Mathematical Relationship between NAD27 and NAD83(91) State Plane Coordinates in Southeastern Wisconsin," published by SEWRPC, December 1994.

> SEWRPC Technical Report No. 35, "Vertical Datum Differences in Southeastern Wisconsin," published by SEWRPC, December 1995.

With these reports, the user is provided a means by which spatial data can be moved from one datum to the other while preserving the integrity of data being converted.

However, during preparation of the aforementioned reports, it became obvious that larger issues involving new technology, emerging uses of spatial data and changing practice also needed to be addressed. This report identifies some of the factors affecting the collection, processing, manipulation, storage and use of spatial data as implemented in a geographic information system (GIS) data base. Those factors include, but are not necessarily limited to:

- Modern spatial data collection systems (Global Positioning System (GPS) instruments, photogrammetric mapping instruments, and conventional total station instruments) make three-dimensional measurements while existing practice continues to rely heavily on two-dimensional mathematical models, i.e., conformal map projections.

- With continuing development of computer, mass storage, and remote sensing technologies, there has been a dramatic increase in the importance of the digital spatial data environment. Analog storage of spatial data on maps and photographs remains applicable in some cases. But, while human perception (computer displays and graphical map products) of spatial data is largely analog, mass storage and manipulation of spatial data is increasingly being conducted in a digital environment.

- Spatial referencing to national horizontal and vertical datums is well-established for reasons of stability, compatibility, uniqueness, standardization, and availability. However, with readjustment by the Federal government of both the horizontal and vertical datums during the 1980s, and subsequent datum refinements, some of the advantages of shared spatial data are eroded by loss of compatibility and the costs of data conversion. To avoid that loss in value, additional attention must be focused on the needs of local spatial data users whose concerns are really more closely related to the stability and quality of local coordinate differences.

- The quality of control survey data and the integrity of datum conversions were prime considerations in SEWRPC Technical Reports Numbers 34 and 35. National Map Accuracy Standards have served and continue to serve as a mechanism for judging the quality of spatial data in an analog environment. In the digital environment, positional tolerance, standard deviations, and error propagation are concepts that can be used to judge the quality of digital spatial data.

This report looks at the broader issues from a global spatial perspective with attention focused on the impact to the local user. Many existing practices are the result of using computer technology to digitize an analog process. Productivity gains are impressive but the end user is still left, for example, with grid scale factors and other features imposed by use of a two-dimensional conformal mapping model. The approach taken in this report is to start with the overall concept of digital spatial data in a three-dimensional environment, to consider the technology used to collect and store spatial data, to incorporate existing practices as applicable, and to accommodate the perspective of the local user at the simplest possible level without sacrificing geometrical integrity. And, recognizing that spatial data are considered to be a capital asset, an underlying objective is to preserve the value over time of the spatial data.

CHARACTERISTICS OF SPATIAL DATA

The term "spatial data," as used in this report, is defined as specific information about the location of a point. An infinite number of general location choices is available such as North America, Wisconsin, Waukesha County. More specific choices include linear referencing (milepost 57.6 along Interstate 94), U. S. Public Land Survey System aliquot parts (NE ¼ Section 12, Township 8 North, Range 18 East), or tax parcel (Lot 32, Greenacres Subdivision). Although each of the foregoing examples is useful in a given context, the term "spatial data," as used in this report, relates to a location mathematically defined by a coordinate system. In the past, plane rectangular two-dimensional systems have been used to represent local features on portions of the earth's surface. In cases where flat earth assumptions were no longer applicable, the curvilinear coordinate system of latitude/longitude was used to identify specific locations. In either case, whether using plane coordinates or latitude/longitude coordinates, elevation above (or below) sea level was used if and when a three-dimensional location needed to be defined.

The two-dimensional map has evolved as the medium for recording the relative location of objects or points on or near the earth's surface. A topographic map with contour lines is an analog entity that records the three-dimensional location of identified features.[1] A map is confined to showing the spatial relationships of points as viewed from a fixed perspective—that chosen by the map maker or cartographer. No other views of the same data are

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[1]*The term "analog" has a number of general meanings given by common usage, dictionaries, and also a number of specific meanings given by usage in such fields as electrical engineering, photogrammetry and remote sensing, surveying and mapping. Within the context of this report, the term analog is used primarily to describe data and information portrayed in a graphic form on a printed map or displayed on a computer screen. The term "digital" is used to describe spatial data stored in a computer data base in numeric (digital) form. Analog and digital data are both used extensively and need to be accommodated within current and evolving practices of spatial data management. Practices for using spatial information rely heavily upon digital-analog conversions. The challenge is to store spatial data efficiently in a digital form which can be most readily processed and converted to other useful forms, both analog and digital.*

permitted or accommodated without making a new (different) map.

With computer technology, however, the display and other derivative uses of digital spatial data are limited only by the imagination of spatial data users. Computer programs are already available for digital "fly-throughs" in which the user can navigate a virtual model of an area represented by a digital spatial data set. A critical link for spatial data users is the physical connection between the virtual model and the real world. That link which should consist of a mathematically rigorous and efficient connection is defined in this report as a three-dimensional global spatial data model (GSDM).

MANAGING SPATIAL DATA

Given that the value of spatial data is recognized as a capital asset, the management of spatial information to preserve its value becomes important. Of the many issues to be considered in this respect, the use and manipulation of digital spatial data in what has historically been an analog environment deserves careful examination. In the past, spatial data have been largely stored in map form. Typically, survey measurements are obtained, either by field surveys or aerial photography, and the information is compiled into an analog map. The analog map is variously used to portray, for example, the boundaries of a tract of land; to show the location of a proposed development; to identify locations of wetlands and floodlands; or to navigate from one location to another. The point is, many communities have made significant investments in spatial data which are stored on analog maps in many locations. Moving that information into digital data storage within the context of a simple, well-defined, three-dimensional spatial data model is one way of preserving the value of existing information.

The digital equivalent of spatial data takes many forms. One classification is a distinction between disposable and archival digital spatial data. For example, the instant location of a GPS receiver is displayed as a set of digital coordinates which change as the instrument is carried from place to place. There is no analog interface and, if the data are not stored, the digital spatial data are used in real time, say for navigation, and discarded. In another scenario, digital spatial data are used with an analog interface in which a computer graphic display shows an instant location superimposed upon a background of local features plotted from information stored in a digital data base. Landing an airplane equipped with a GPS guidance system is an example where, in addition to visual observation through the wind screen, the pilot can watch the approach on a changing analog display. The digital spatial data used to create the display are not needed after the aircraft has landed and may be discarded. Landing an airplane is used as an example to make the further point that, although digital spatial data may be used and discarded, the accuracy and reliability of the data are critical. There is a high correlation between the quality and value of digital spatial data even in cases where the data are used briefly and discarded.

Archived digital spatial data, such as those stored in a parcel-based land information system (LIS), are the focus of this report. In this mode, spatial data can be tested carefully to prove their integrity before being stored in an electronic data base. Once stored, the digital spatial data can be recalled as often as needed for specified purposes and the value of high quality digital spatial data can be utilized repeatedly as opposed to the one-time use of disposable digital spatial data. A digital LIS is more versatile than an analog map because:

- The same data can be readily duplicated electronically and shared with others without changing its inherent qualities.
- The LIS is the repository of the primary spatial data which can be used and reused many times to make analog maps as needed to serve specific purposes. The point is the analog map in this environment becomes essentially "disposable" as it can be used and discarded without consequence.
- Subject to the type and quality of data available, a large variety of analog maps and charts can be made with little or no restriction on plotting scales or perspectives.
- Digital spatial data files are more readily adaptable to cataloging, organization, and storage than analog maps. Defined attributes of digital spatial data can be selected and/or analyzed using relational data base tools to support a more sophisticated decision-making process than can an analog map.

But, whether digital or analog, disposable or archivable, the benefits of having and using spatial data should be greater than the cost of collecting and compiling it. If the information is insufficient, inferior, incompatible, or defective in some way, the benefits of having and/or using it are compromised accordingly.

Several axioms are:

- The value of existing spatial data is enhanced and preserved to the extent the data are compatible with other spatial data and as the integrity of the data can be efficiently documented, tested, and proven.

- The value of digital data is realized through the use of derived analog maps and of displays.

- The cost of accurate spatial data can be best amortized by exploiting multiple applications and uses for the same information.

- Preparing analog maps from an adequate digital spatial data base can be much more efficient than making the same map from analog field survey data.

- Information from the same digital spatial data base can be sorted many ways and used repeatedly to make an infinite number and/or variety of maps.

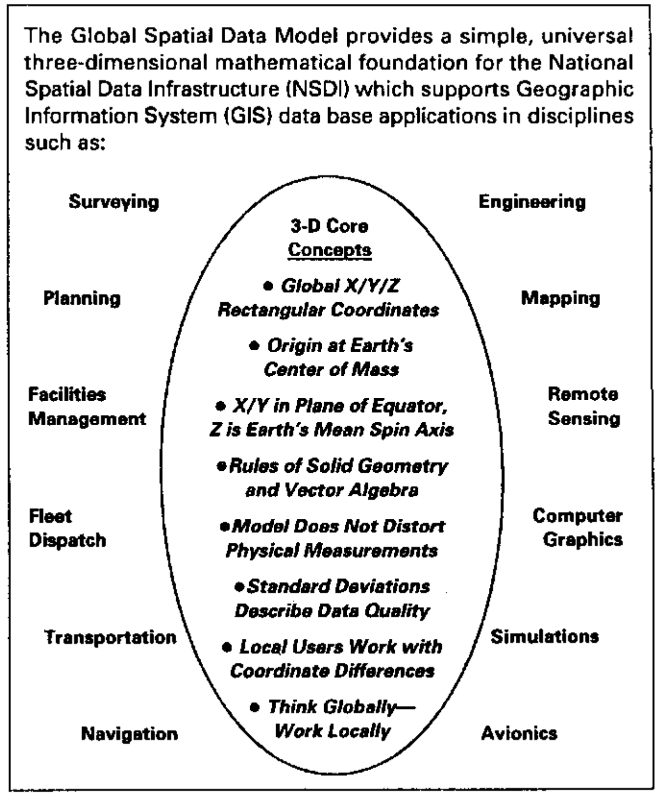
- Limitations on the use of spatial data may be obvious in some cases, but not recognized in others. A free-hand sketch of a river crossing clearly will not support the proper engineering design of a new bridge and a plat showing property ownership does little to delineate the existence of a flood hazard. In less obvious cases, bad decisions may be made because limitations on the quality of available spatial data are either not known or are not recognized. Worse yet, known limitations may be ignored because they are not provable or documented.

DESCRIPTION OF THE GLOBAL SPATIAL DATA MODEL

The issues highlighted thus far are intended to be addressed with a Global Spatial Data Model (GSDM) which can be used by many disciplines (see Figure 1 and Burkholder, 1977). The GSDM has two components—a functional model and a stochastic model. The latter is described further in Appendix A. The functional model is the geometrical basis of the GSDM and may be described as an earth-centered, earth-fixed coordinate system which is a right-handed three-dimensional X/Y/Z rectangular cartesian coordinate system with the origin located at the earth's center of mass. The X/Y plane is coincident with the earth's equatorial plane and the X axis piercing the Greenwich Meridian at the equator. The Z axis is coincident with the mean spin axis of the earth. Geometrical integrity is preserved throughout by using universally applicable rules of solid geometry and vector algebra.

Figure 1

GLOBAL SPATIAL DATA MODEL—GSDM
(A Universal 3-D Model for Spatial Data)



The stochastic model portion of the GSDM, while optional in this context, is well-defined for all possible levels of use. If the stochastic model is not used, the variance/covariance matrix associated with the geocentric X/Y/Z coordinates of each point is assigned "zero" default covariance values. Using zero covariance values does not affect use of the functional model except that quantities having no covariance data available are used as being "exact." If covariance or standard deviation values are available, they can be used and the value of the data is enhanced accordingly. Using standard error propagation techniques, the standard deviation of a point position can be described in the local east/

north/up coordinate system and the standard deviation of any direction and distance between points is part of the inverse computation. The stochastic model and its use is the same whether used with very precise data (small standard deviations) or approximate data (large standard deviations). Precise data can be intermixed with approximate data without consequence with the understanding that the standard deviation of derived results is based upon the standard deviations of the data being used. Furthermore, there is no requirement that the standard deviations of various vector components be the same. Implications of using the stochastic portion of the GSDM along with the function model portion are:

- The functional model is applicable whether or not the stochastic model is used.

- The user has control over, and responsibility for, the quality of the data. The model accommodates the user's judgement about the quality of data going into the data base.

- The model—both functional and stochastic—is fully three-dimensional and makes no distinction between horizontal and vertical. In fact, the two are seamlessly combined into one integrated global data base. Local perspective is preserved by working with coordinate differences relative to a user selected latitude/longitude/height location.

The GSDM is an umbrella concept which incorporates, but does not displace, existing practice. It conveniently accommodates past surveying and mapping practice, digital—as opposed to analog—data storage, and new, high technology, spatial data collection systems. The GSDM provides modern cartographers and GIS users unlimited options with respect to ways spatial data can be displayed, viewed, plotted, or printed. Specific features of the GSDM include:

- Spatial data are stored in digital form in an electronic computer data base.

- The physical location of each point stored in the data base is represented by the X/Y/Z geocentric earth-centered, earth-fixed rectangular coordinates. Display of the coordinate location can be in any user-defined or selected system.

- A local perspective of all points defined in the data base is immediately available. The user has the option of selecting any "standpoint" and looking at the relative location of any other "forepoint" desired. The distance reported is the same ground level horizontal distance used in plane surveying. The direction is the azimuth from north relative to the meridian through the standpoint.

- Once measured, computed, checked and verified, the primary definition of each spatial point anywhere on the earth or within the "birdcage" of orbiting GPS satellites, is expressed in the X/Y/Z geocentric rectangular coordinates of the point along with a statement of standard deviation of each rectangular component. Standard deviations in local east/north/up directions are computed from stored geocentric covariance values.

- Manipulation of three-dimensional rectangular coordinate data follows well-established rules of solid geometry. Vector algebra is universally applicable and 3-D networks can be adjusted using a linear model. Additionally, computational integrity is enhanced by using algorithms designed to work with local three-dimensional coordinate differences.

- Elevations are not obtained directly from stored X/Y/Z coordinates, but are computed as the difference between ellipsoid height and geoid heights. The ellipsoid height is obtained from the X/Y/Z coordinates and the geoid heights are obtained from an appropriate geoid model. If standard deviations for both ellipsoid height and geoid height are known, the standard deviation of the elevation is readily determined. In the past, reliable geoid heights have not been available but the National Geodetic Survey (NGS) provides geoid models which can be used to find geoid heights at any location in the United States. As better geoid models become available, it will become possible to obtain elevations (orthometric heights) as good as those obtained with differential leveling. The GPS test described in Appendix D provides determination of NGVD29 elevations for four points, all of which agreed within 0.12 foot of the known second-order accuracy elevation. More impressive is the fact that elevation differences between the four points agreed within first-order specifications for conventional differential leveling and that the elevation difference between two benchmarks located one-half mile apart was the same as the published elevation difference.

- With its global scope, simple definition, and universal use of solid geometry equations, the GSDM enhances decentralization of spatial data usage and application. It models the local view of the world while at the same time preserving a global geometrical integrity. Converting from one datum to another, or from one coordinate system to another, is simplified by using the global spatial data model as a universal intermediary. Each user need only be concerned with the specific relationship of the datum or coordinate system of interest with the standard.

The GSDM does not involve invention of new mathematical principles or applying new science. Rather, it is a formal description of interrelationships which exist between concepts already defined and being used in various contexts. This study describes how combined concepts of the GSDM can be used to manage and use digital spatial data more effectively, especially from any "local" perspective. The functional model portion of the GSDM is illustrated in Figure 2 and the algorithms are listed in Appendix B-1. The GSDM accommodates three-dimensional spatial data currently defined by geodetic and/or geocentric coordinate systems. Other 3-D data defined by local, project datum, and/or state plane coordinate systems can be converted to and entered into the GSDM subject to acceptable definition of the two-dimensional horizontal coordinate system and acceptable knowledge of the geoid height need to provide specific connection between elevation and ellipsoid heights needed for true 3-D.

Algorithms for using the stochastic model portion of the GSDM are listed in Appendix B-2. The standard error propagation procedures, as described in Chapter 4 of Principles and Techniques of Propagation, by Mikhail (1976), are applied to the functional model equations for each of the listed computational steps.

Regardless of the mathematical model being used, spatial data are determined as the result of a measurement process and the uncertainty (standard deviation) of each physical measurement is the basis for error propagation through the stochastic model. Length is the fundamental physical quantity most closely related to spatial data. Other fundamental quantities such as time, current, mass or temperature are used indirectly to obtain spatial distances. The stochastic model and associated error propagation used in conjunction with the 3-D GSDM accommodates measurement of all fundamental quantities and subsequent computations performed using them. At the most elemental level, the measurement of a physical quantity is independent of other variables, in which case, covariance values in the covariance matrix are zero. If correlation exists between computed values based upon the independent measurements used as variables in the functional model, the correlation is automatically computed in the stochastic model through the error propagation process. Derived quantities, such as distance, area, volume, or direction, are of particular importance and the accuracy of the derived quantities is given by their standard deviations.

In summary, the 3-D spatial model stores the rectangular, geocentric X/Y/Z coordinates of each point in the earth-centered, earth-fixed coordinate system. Those rectangular coordinates are manipulated efficiently using rules of solid geometry and vector algebra. Existing data bases are capable of storing large quantities of such digital spatial data, and visualization routines are available by which a given set of spatial data can be viewed in many ways from any desired perspective.

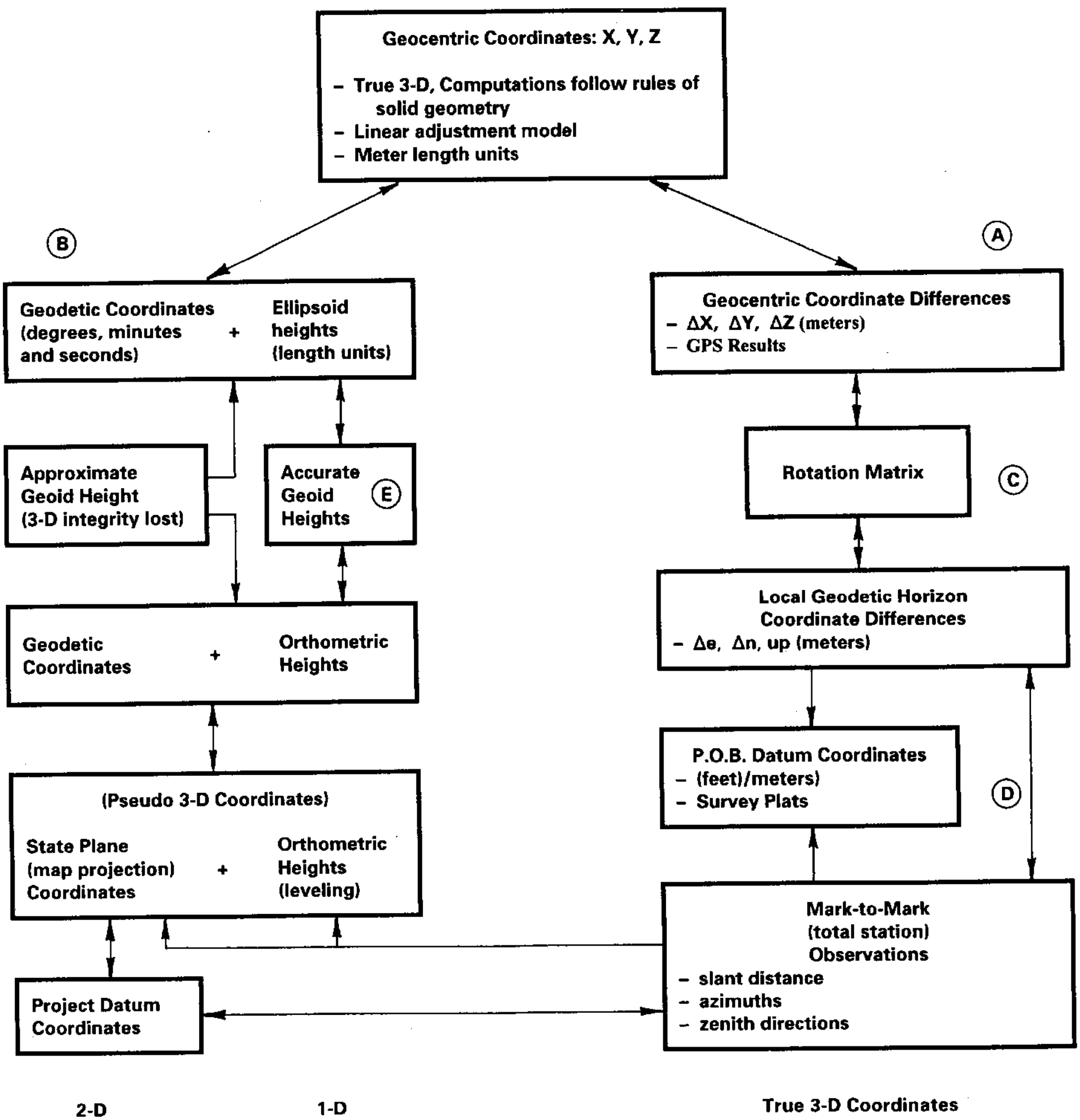
The 3-D spatial data model also stores the stochastic model values as the covariance matrix for each point in the geocentric X/Y/Z system. Information about the standard deviations of any spatial data component in any direction is derived by applying the appropriate error propagation routine to the data and algorithms. This provides the user immediate answers to the questions, "How good are the data?" and "How accurate is (what is the standard deviation of) the quantity computed from coordinates stored in the data base?"

DATA BASE CONSIDERATIONS

Characteristics of the GSDM are such that, within several constraints, it can be implemented in a decentralized environment where the user has a great deal of latitude with respect to use of the data. Given that there is one point per record in a data file, it is important to identify the number of fields required to support data exchange which fulfills both the functional and stochastic elements of the GSDM. The constraints to insure the compatibility of points are that: 1) the location of a point is defined with metric geocentric X/Y/Z coordinates; and 2) that the stochastic model (information on the quality of position), if one is used, is stored as

Figure 2

DIAGRAM SHOWING RELATIONSHIP OF COORDINATE SYSTEMS



the geocentric covariance matrix for the point. Since the 3 x 3 covariance matrix is symmetrical, six fields are required to store the upper (or lower) triangular matrix values. Three fields are needed for the X/Y/Z coordinates, making a total of nine fields per point. As a minimum beyond that, an integer field is suggested for a point identification number and a literal character field of nominal length should be reserved for station name or brief descriptor. Other fields containing attribute information or codes for relational data base storage/retrieval can be allocated as deemed appropriate for a given application. Those fields would be beyond the nine fields required (or the 11 fields suggested) as a minimum for digital spatial data exchange.

In addition to minimum requirements for compatible exchange of spatial data from one data base to another, the following is a description of several levels at which a 3-D data base might be implemented. The basic model definition and principles are identical in each case. The difference is the level of integrity or administrative control. Several levels may exist in the same organization for different purposes. Again, the data (coordinates and covariances) are standard at all levels.

- Military/scientific. The ultimate in precision, quality control, administration and use will serve a special user community and can exist independent of civilian/commercial uses.

- National datum/civilian. Control information in the National Spatial Datum System would be administered in conformance with good science and needs of a broad civilian user community. It could enjoy seamless integration with the military system subject to differences in ultimate precision. For example, the highest level worldwide might exist at 0.001 ppm, while the next (national) level is supported at the 0.02 ppm level. Commercial navigation and mobile mapping activities, while normally supported by the civilian data base, would be compatible with use at other levels, both higher and lower.

- State/professional. Controlled and administered by Federal or state agencies, the data base would support the activities of many disciplines, whether for mapping, engineering, GIS data management, land surveying, or other spatial data activities.

- County or municipal/proprietary. A local 3-D data base can be constructed by anyone willing to build and support it. The procedures for building and using a 3-Ddata base are identical at all levels and the integrity of the data base is the responsibility of the owner.

The integrity and value of a 3-D data base at any level will be a function of the standards and specifications enforced for quality control of data entered into the data base. Standard deviations associated with all input data will provide each subsequent user with information necessary to judge whether the data obtained from the data base is appropriate for or will support the intended use. Many users will glean control data from the appropriate data base and build a local data base containing all data types; primary, observed, secondary, design, derived, and, perhaps, archived consumable data (Burkholder 1995). Beyond meeting minimal requirements for compatible exchange of digital spatial data, local users have great latitude with regard to the manner in which spatial data are used. Local users maintain local administrative control.

The following three levels or types of data should be considered by the Southeastern Wisconsin Regional Planning Commission in plans to build a comprehensive 3-D data base.

- Primary control data could be in the form of 3-D positions for the High Accuracy Reference Network (HARN) points published by the NGS and established according to NGS standards. NGS imposes rigorous quality control standards on the data going into their data base. Although not a large number of points, this primary level should serve as the basis for all other 3-D positions established throughout the SEWRPC Region. Users with very high accuracy needs would rely on these data exclusively as control for subsequent spatial data measurements.

- The SEWRPC has, over the years, carried out many control surveys of high quality. The resulting control survey data can be added to create a file of densified control for use by SEWRPC and others as primary control for all but the most exacting applications. As a logistical consideration to keep the file size manageable, points entered into this file would be expected to pass rigorous quality assurance tests. This level of data base would serve as the control source for lower-level "production" activities.

- At the production level, many users will create many files containing digital spatial data with a focus on specific applications. While such coordinate positions may be compatible within a given project, associated standard deviations can be used to determine the extent to which project data may be compatible with coordinate data obtained from other sources. For example, survey data from other agencies might be combined with SEWRPC data in a given area at a specified level of accuracy. Standard deviations and documented datum differences provide a basis for sound decision making in this regard.

The underlying assumption is that various users will be able to rely on valid determinations of standard deviations. Although standards and procedures for the higher levels are published and applicable, as a practical matter, they are too onerous in existing form to be imposed upon spatial data derived at local levels. Additional effort is required to identify efficient quality control procedures and to develop the tools for tracking standard deviations for lower-level data. The trade-off will be to find the right balance between the effort required to establish reliable standard deviation values and the economic value supported by the proven integrity of the data. The point here is that the 3-D spatial data model will support development and application of consistent procedures for establishing and proving the integrity of digital spatial data, regardless of the discipline needing or using the data.

PROCEDURES FOR BUILDING A 3-D DATA BASE

Regardless of the type of data base being constructed, the principles are the same. The primary difference in the types would be in the highest level of precision supported or in the administrative control over access and use of the data base. Subsequent logistical considerations, such as number of users or minimum level of positional tolerance accepted, could also create differences in features or details at various levels of implementation. But, the following underlying principles are applicable at each level:

- A fundamental tenet for a data base built at any level is that primary data within that data base must be able to support the highest level of expected use. It is appropriate to hold (assign a zero standard deviation) to primary data having a proven accuracy at the 5 to 10 parts per million level in a data base intended to support local spatial data manipulations at the 20 to 50 parts per million level, but not to use the same data base as control for a global positioning control survey system densification project in which positions are established at the 1 to 10 parts per million level.

- Control data and measurements (along with appropriate standard deviations), added to the data base must meet quality control standards and specifications adopted for the level of intended use for the data base. The standards and specifications for new measurements and quality control procedures need to be published, understood and enforced. Otherwise, integrity of the data base will suffer.

- Data submitted to a data base which fails to meet given quality control standards could still be included (subject to administrative edict and available storage space) if properly qualified by their larger standard deviations.

- Legitimate users of any 3-D data base may download information from same and use the data as primary data in a lower-level data base subject to meeting stated standards and specifications of target data base.

The NGS verifies the data before it is released for public use. Data added by others can also be quite useful and can be very valuable. The challenge will be to assure the appropriate standards of data acceptance are agreed upon and used. Data with a larger standard deviation can be useful but, with the standard deviations readily available, better decisions can be made by the end user about subsequent use of the data.

3-D DATA DEFINITION/INPUT

Digital spatial data added to a 3-D data base are either converted from the archived information published for existing points or they are newly surveyed points not yet part of an existing data base. Existing points already defined in three dimensions includes high-level control points established by Federal/State agencies and private sector firms and organizations in addition to thousands of points established by the Commission. Regardless of whether the points are defined on the NAD27 or NAD83(91) (for horizontal) or NGVD29 or NAVD88 (for vertical), algorithms and procedures exist for the SEWRPC area whereby reliable 3-D positions and standard deviations of those positions can be determined and included in a 3-D data base. As explained later, accurate geoid height is the weakest link in the process and may need to be addressed through a separate geoid height study. The important point here is the 3-D model is appropriate and, even with current uncertainty in geoid heights, 3-D positions can be determined and used.

The second general source of digital spatial data is the measured location and uncertainty of new points added to a data base. The coordinates of the new points are computed according to the geometry of the functional model and the positional tolerance (standard deviations) of the new positions is determined by the accuracy of previously defined points and the measurement precision of vector components added to the data base. Examples include:

- GPS baseline vector components. These geocentric ΔX/ΔY/ΔZ coordinate differences are added to previously established (or simultaneously determined) network points.

- Local Δeast/Δnorth/Δup components obtained from total station observations. Given the height of each instrument setup and the height of the reflector shot are duly recorded, the resulting terrestrial components (and standard deviations) are efficiently rotated into geocentric components by a rotation matrix and can be done easily on a hand-held calculator. For precise surveys, a correction may need to be applied for deflection-of-the-vertical effects, but generally, 3-D conventional traverses are run between 3-D control stations established by gravity-independent GPS observations and the need for such corrections will be infrequent. In extreme cases, a deflection-of-the-vertical and a polar motion correction will both be needed.

- Photogrammetric mapping operations. With the location of each exposure station defined in three-dimensional space by GPS, the three-dimensional spatial differences between points on the ground are computed from the intersection of rays at well-defined objects in adjacent photographs independent of earth curvature. The elevation of each defined point (if and when needed) will be readily derived from its geocentric coordinates and an accurate geoid height at that location. Alternately, this technique might be used in conjunction with known elevations to improve the quality of geoid heights in a given area. Other interpolation techniques may also be appropriate.

Management of spatial data storage, access, and use is well established in many operations. A formal procedure for testing/proving the quality of data is the responsibility of a designated office for putting the data into the data base. Once there, the data are available for all users having "read" access to the file.

USE OF 3-D DATA

In a general sense, spatial data are used:

- As a reference for the location of a feature or object. When used in this mode, geometry is of secondary importance as the overriding characteristic is the local, regional, or global uniqueness. Geocentric coordinates are globally unique and could be used efficiently as such, but they can also be expressed as latitude/longitude/height, Universal Traverse Mercator coordinates/elevation, state plane coordinates/elevation, or any other locally defined system. Understandably, global uniqueness is lost when using a map projection (unless a zone is also specified) but in some cases, regional or local uniqueness is entirely sufficient.

- As pairs of points. In many cases, the overriding question to be answered is the local direction and distance between two points. That answer is obtained directly from the geocentric coordinate differences rotated to local plane surveying components Δeast/Δnorth/Δup (Δe/Δn/Δu).

  1. Horizontal distance is the local tangent plane distance $\sqrt{(\Delta e^2 + \Delta n^2)}$ land surveyors have been using for generations and no correction need be applied for elevation or scale factor. It is identical to HD(1) in Burkholder (1991).

  2. In plane surveying, direction is obtained as arctan (Δe/Δn). But, a careful choice will need to be made. Either choice is legitimate and each may be preferred under different circumstances. The difference is whether the reference meridian is counted as going through each instant station or held as being that through

some local "master" station. Briefly, the options are:

a. The local direction between the two points concerned related to true north is computed as the arctan ($\Delta e/\Delta n$) as in plane surveying. In one sense, the model is too precise because the direction from the second point back to the first point will not be exactly 180° different due to convergence of the meridians. When used directly, the 3-D model will always give the true azimuth as reckoned from the meridian through the instant station.

b. Another choice is to bring points out of the data base as related to some master Point-of-Beginning (P.O.B.) Station for a given project. The local latitudes and departures of any point relative to the master station can be treated as local plane grid coordinates. In this environment, the procedures are identical to local plane surveying practices. The specific requirement is that the P.O.B. station must be identified on each survey plat.

3. The Δu component is the perpendicular distance from the local tangent plane to a point at the other end of the line. Curvature and refraction will be needed to obtain a true difference in elevation if computed according to conventional plane surveying practice but, given an accurate geoid height at each station, elevation at each station can also be obtained directly via the 3-D spatial data model.

- To make maps. Cartographers have for centuries wrestled with the issue of portraying a round earth on a flat map. As analog graphical constructions have given way to mathematical manipulation and visualization of digital data using powerful computers, the number of ways digital spatial data can be portrayed is limited only by the imagination of modern cartographers.

- To define digital terrain models. Enormous computer files of digital spatial data have been created and are being used for many purposes. In some cases, it will be economical to convert such files to the 3-D GSDM, but in other cases, just defining the relationship to convert the data from one file to another will suffice. In a broad sense, each user is only responsible for the relationship between the local data base and the GSDM. In many cases, the relationships are already well-defined.

The 3-D GPS test example given in Appendix D illustrates several of the described uses. Once points are defined in the data base, they can be printed out in any coordinate system desired by the user. A well-defined mathematical relationship must exist between the geocentric coordinates and the desired system. The print-out in Appendix D shows a listing of the geocentric coordinate values opposite circled letter Ⓗ. While these values are unique, few people can visualize or relate to those coordinates in a meaningful way because we have not learned to think in terms of the geocentric values. Neither is it required. With a standard simple calculation, the location of any forepoint with respect to a user selected standpoint can be computed. And as shown in section Ⓘ of Appendix D, the familiar geodetic latitude/longitude/height coordinates are listed along with the geocentric values. Additionally, the covariance values for each point are listed, both in the geocentric reference frame as well as the local frame. The standard deviation for each coordinate is the square root of the variance given on the diagonal of each matrix. The GSDM readily accommodates it, but there is no correlation in the point covariances because the HARN control points were assumed to be errorless and the standard deviations of the GPS vector components, although different from one vector to another, were identical for the geocentric components on each vector.

Keyed sections Ⓙ, Ⓚ, Ⓛ, Ⓜ, and Ⓝ in Appendix D show various inverses between points of the 3-D test. In each case, the expanded listing is given for the point at each end of the line. Note, too, the local standard deviations are listed for each endpoint. Between endpoints, the geocentric coordinate differences, the local coordinate differences, and the plane coordinate inverse is given in each case. Furthermore, the computed standard deviations for the derived inverse quantities are also given.

The local vertical component in the 3-D example in Appendix D is ellipsoid height or, in the case of each inverse, the difference in ellipsoid height, and should not be confused with elevation. As shown at circled letter Ⓔ in Figure 2, the difference between ellipsoid height and orthometric height (elevation) is geoid height. A specific point is that the local "up" component of a spatial vector is the perpendicular distance from the forepoint of the vector to the tangent plane through the standpoint of the same vector. Geoid heights are described in more detail in the next section.

GEOID CONSIDERATIONS

Geoid height, the difference between ellipsoid height and elevation, is critical to successful implementation of a three-dimensional spatial data model. In the past, surveyors, mappers, and engineers have relied upon separate horizontal and vertical datums as the basis for spatial referencing. Now, with the advent of three-dimensional measurement systems, computer data bases, and electronic manipulation of spatial data, the advantages of using a combined three-dimensional spatial data model are coming to the fore. But human perception of "vertical" is a given and use of a computerized digital spatial data base must accommodate use of elevations.

Mean sea level—that is, the geoid—is an intuitive vertical reference for elevation and has been so used for generations. But the reality is, no one has been able to find the precise location of the geoid (mean sea level) commensurate with the quality of measurements being made. As a consequence, mean sea level has been effectively abandoned as a reference surface by the scientific community. The GSDM is well defined in three dimensions and readily supports computation of an elevation derived from ellipsoid height and geoid height. Ultimately, that means the earth's center of mass becomes the primary vertical reference. The critical question for surveyors and other spatial data users is at what point and under what circumstances can ellipsoid heights and geoid heights be used to derive an elevation which is as good as an elevation based upon mean sea level and differential leveling.

Recognizing the futility of finding true mean sea level, in 1973 the National Geodetic Survey changed the name of the vertical datum from the "Mean Sea Level Datum of 1929" to the "National Geodetic Vertical Datum of 1929" (NGVD29). The change was in name only and no elevations were changed. But, the change did remove the implication that one could start from a published bench mark along a sea coast, run differential levels until one reached a "zero" elevation, and call that mean sea level. Although NGVD29 elevations are still referenced to years of record at 26 tide gages located along the North American coast, the datum origin is acknowledged to be arbitrary.

Another step in treating mean sea level (the location of the geoid) as a derived quantity occurred with readjustment and publication of the North American Vertical Datum of 1988. In that readjustment, the interior level loop misclosures demonstrated a higher level of geometrical consistency than the tide gage derived differences. Therefore, the decision was made to choose an elevation for one arbitrary bench mark, Father Point–Rimouski, Quebec, Canada, and to publish elevations of all other bench marks in the network with respect to that one point. The remaining tide gages are useful for documenting continuous water height, but are no longer used to define mean sea level.

High precision scientific applications are appropriately supported by well-defined models. But the local spatial data user is far more concerned with local elevation differences (and changes in the differences) than in knowing more specifically what the distance (orthometric height) is from a local benchmark to a reference surface such as the geoid or ellipsoid. The GSDM enjoys geometrical integrity in all three dimensions and can be used to monitor local vertical differences regardless of which vertical datum is used. And, as described in the next section, with the addition of proven modeling tools, the GSDM accommodates new measurement technologies which can be used to determine elevations more efficiently than with conventional differential leveling.

ELEVATIONS FROM THE GSDM

Accurate geoid heights are required in order to obtain good elevations from the GSDM. Although costly, the most precise way to determine the difference between the ellipsoid and the geoid is to determine ellipsoid height very accurately with GPS observations and compare it with a precise elevation for the same point determined using high order differential leveling. With accurate geoid height known, elevation can be obtained from the GSDM. Again, although quite accurate, this method is costly. Other methods involving estimation and interpolation are generally less expensive and, depending on circumstances, can be quite accurate.

It is well known that the changing height of the geoid (geoid undulation) is a consequence of gravity and its variations. Given that a level surface (the geoid) is always perpendicular to gravity, the implication is that the geoid must be a continuous surface and that the shape of the geoid is related to both the direction and strength of gravity.

If the direction of gravity were known at every location, it would be possible to infer the shape of the geoid and, given a starting value, to calculate the distance between the ellipsoid and the geoid at all locations. Although the observed direction of gravity (the plumb line) is used as a reference for other observations, it is only with great effort that the direction of gravity at a point can be observed accurately with respect to the ellipsoid normal.

Separately, with the strength (magnitude) of gravity known at every point, it is possible to compute geoid heights based upon a starting value at some point and a complete set of gravity measurements. Although instrumentation exists whereby the magnitude of gravity can be measured, it is impractical to measure gravity accurately enough at a sufficient number of locations to compute precise geoid heights. However, with a much smaller number of representative gravity measurements, it is possible to model the shape of the geoid in an area quite well.

During the 1980s, staff at the Ohio State University (OSU) conducted extensive research on computing geoid models from gravity measurements. Using an OSU model and data from various sources, the National Geodetic Survey has prepared and released several computer programs that can be used to compute estimates of geoid heights at all locations throughout the United States. The first such program was GEOID90, followed by GEOID93 and, more recently, by GEOID96. Appendix C contains a copy of the "readme" files which accompany the GEOID96 program. Appendix C also contains a plot of GEOID96 results computed on a 10,000 foot grid over the seven-county SEWRPC area. The isoline numbers represent the modeled distance between the mathematical ellipsoid surface and the NAVD88 geoid and are negative because the geoid lies below the ellipsoid throughout the Region. Another program available from NGS, VERTCON, is required to determine the additional difference between the NGVD29 geoid and the NAVD88 geoid. See SEWRPC Technical Report No. 35, "Vertical Datum Differences in Southeastern Wisconsin."

These geoid modeling programs are valuable, but there are limitations associated with their use. When provided an NAD83 latitude/longitude location within the United States, GEOID96 will return a modeled geoid height in meters to three decimal places. That is impressive, but the answer cannot be relied upon at the millimeter level. The accuracy is stated in the GEOID96 readme file to be more nearly in the 3 cm (one sigma) range. However, knowing that the shape of the geoid can be modeled with greater certainty than the absolute geoid height, it is possible to use the difference in geoid heights between points along with changes in ellipsoid heights obtained with GPS to determine elevation differences with an accuracy approaching that provided by differential leveling.

The procedures used and the findings of the 3-D GPS test reported in Appendix D may be summarized as follows:

- GPS receivers were set on four stations and data collected simultaneously. Two of the stations were high accuracy (HARN) stations located about 25 miles apart. The other two points were adjacent U. S. Public Land Survey System corners located approximately midway between the HARN stations.

- The 3-D positions of the two U. S. Public Land Survey System corners were computed separately from the two HARN stations with excellent agreement. Standard deviations for the corners were computed using the stochastic portion of the GSDM assuming the HARN stations to be errorless and using the reported standard deviations from the GPS base line computation report.

- Second-order NGVD29 elevations were determined for all four points using conventional second-order, class II differential leveling.

- Estimates of the difference between NGVD29 and NAVD88 at each of the four points was obtained using the NGS program VERTCON (see SEWRPC Technical Report No. 35) and modeled geoid heights at each station were obtained using NGS GEOID96.

- Direct estimates of the NGVD29 elevations at all four points based on the published 3-D position of the HARN stations, GPS observations, the VERTCON-derived difference between NGVD29 and NAVD88, and GEOID96 agreed within 0.112 foot as compared with the known elevations determined by differential leveling.

- Elevation differences determined using the GPS and related data agreed with the known elevation differences determined by differential leveling within Federal geodetic control survey specifications for conventional first-order leveling. While this test alone should not be used to claim GPS can be used to establish first-order elevations, the results are impressive and provide justification for further tests.

POSSIBILITY OF AN ADDITIONAL GEOID STUDY

The foregoing example demonstrates the value of good modeling and the kind of results which can be obtained using existing data and models. Two points to be made are that the existing GEOID96 model and VERTCON program can be used with appropriate GPS observations to obtain:

- NGVD29 elevations within about 0.1 foot.
- Elevation differences, regardless of the datum, with an accuracy approaching first-order.

This raises the issue as to whether or not it is necessary or prudent to invest in an additional geoid study within the Region with the objective of "calibrating" the GEOID96 model so that reliable elevations on NGVD29 can be obtained directly at a point. Factors relevant to consideration of this issue include:

- The Wisconsin Department of Transportation (WisDOT) may undertake a statewide geoid study in support of its work.
- Milbert (1991), in "An Accuracy Assessment of the GEOID90 Geoid Height Model for the Commonwealth of Virginia," describes the objectives, procedures, and process of a statewide geoid study in which he says, "The external approach to the GEOID90 accuracy assessment was quite successful. Based upon comparisons with GPS and leveling, GEOID90 provides at least 10 cm. accuracy (one sigma) between points spaced at 100 KM, and 1-cm accuracy between points spaced at 10 km." Procedures for conducting a geoid study are well documented.
- Beginning in July 1992, the NGS, in cooperation with the California Department of Transportation (CALTRANS) undertook a study to estimate GPS-derived orthometric heights (elevations) in San Diego County, California. Although the project was thorough and extensive, Parks & Milbert (1995) concluded ". . .we cannot make a definitive statement regarding the accuracy of the research geoid relative to GEOID93."

Before deciding to conduct or to participate in a geoid study covering the seven-county Southeastern Wisconsin Planning Region, the SEWRPC should consider:

- The objectives and cost of a geoid study.
- The procedures to be used and the organization which would conduct the study.
- The data types and sources which would be incorporated in the study.
- The publication and use of result and conclusions of the study.

ISSUES RELATED TO IMPLEMENTATION OF THE 3-D GSDM

Although the fundamental appropriateness of a three-dimensional global spatial data model (GSDM) is well-documented, issues relating to implementation of the global spatial data model need additional clarification. A summary of issues deserving addition discussion includes:

<u>Datum Refinements</u>
Once the local spatial position and standard deviations of a point are defined in the GSDM, its continued use and value should be immune to loss due to refinements in the underlying geodetic datum. Acknowledging that future determination of parameters for the International Terrestrial Reference Frame (ITRF) may be more accurate than those now being used, and acknowledging that arguments may be made in the future for use of an improved set of numbers for the definition of NAD83 based, for example, on a comprehensive readjustment of the statewide HARN's, procedures could be identified which permit scientific and other high-level uses of the refinements without imposing a conversion burden on local spatial data users. It should be understood, in this respect, that changes in local coordinate differences which are greater than standard deviations of the previously used distance will need to be addressed. But, unless the local difference actually changes, local users should remain immune to impacts of inevitable datum refinement. The stochastic portion of the GSDM should provide valuable tools for developing the required policies and procedures.

Elevation/Gravity

In a narrow sense, the GSDM is strictly spatial and is governed by rules of solid geometry and vector algebra. However, the impact of gravity must also be considered with respect to:

- Plumb line referenced measurements. Total station observations (horizontal distances and vertical angles) and differential leveling measurements are referenced to the local vertical instead of the ellipsoid normal. Circumstances—high precision surveys—for which the deflection-of-the-vertical cannot be ignored need to be identified and procedures adopted for applying corrections to the observations.

- Elevations. Local spatial data users rely upon elevations referenced to the geoid to establish slopes and grades. Geoid height is the connection between ellipsoid height and orthometric height and is needed to obtain elevation from the GSDM. In the recent past, significant advances have been made in the collective knowledge of the geoid and current research supports knowledge of the geoid at a level approaching that of differential leveling. And, when very accurate knowledge of a hydraulic gradient is required, dynamic heights such as used for the International Great Lakes Datum are employed. When used with appropriate error propagation procedures, the height (elevation) of a point (both orthometric and dynamic) and its standard deviations will be readily available from the GSDM data base.

Hierarchy of Precision

Spatial data are not exact. The difference between levels of spatial data precision needed by various disciplines and applications needs to be identified. The differences range from approximate data used in navigation and many GIS applications to engineering and land survey quality measurements used in public works projects and in the compilation of cadastral records to very precise measurements used in deformation and crustal movement studies. A comprehensive tabulation of applications, along with detailed procedures showing how the quality of spatial data (standard deviations) is treated in the GSDM, is needed.

Error Propagation

The mathematical theory of variance-covariance propagation is proven and accepted, but additional work is needed to document application of the stochastic model to 3-D spatial data. Consistent methods for handling measurements, errorless (design) dimensions, and derived quantities need to be identified and formalized for the user community. Procedures for consistent application of positional tolerance concepts will be quite useful to spatial data users in many disciplines.

Mean Positions

A premise of using the GSDM is that mean X/Y/Z positions can be adopted for control points affixed to the earth's surface plates and land masses. The following issues have already been addressed to some extent by the NGS but need to be documented in the context of the GSDM.

- Earth Tides. Positions on the earth's crust move vertically a measurable amount throughout the day due to forces of gravity interacting between the earth, moon and sun. Based upon GPS data available from simultaneous worldwide observations, a mean position of given ground stations can be precisely determined. The mean position will support the needs of most national datum users and temporal variations from the mean position can be made available to those needing such information for high precision applications.

- Earthquakes. Catastrophic events which cause sudden changes in local coordinate differences will also affect the adopted mean position for monumented points in affected areas. As is already being done in such cases, epoch data become part of the public record of such local coordinate difference changes. Acceptable procedures need to be specifically identified and adopted.

- Continental Drift/Subsidence/Uplift. Gradual changes in mean positions due to continental drift, subsidence due to fluid and mineral extraction, glacial rebound and tectonic uplift need to be addressed by a policy which recognizes the need for long-term stability in local coordinate differences while accommodating the fact that absolute positions do change. The level at which periodic updates become necessary is related to the level of accuracy needed by the various users. Changes to a published position should not be made unless and until the documented movement exceeds two or three times the standard deviation of the published position.

Mapping and Charting
Cartographers are experts at making maps (an analog product) to convey information about spatial relationships. Historically, maps have been two dimensional and cartographers have dealt with the issues involved in portraying a round earth on a flat map. Advantages offered by the conformal map projection have been exploited since the Mercator projection was invented in the early 1500's. The conformal projection offers excellent mathematical integrity for two-dimensional relationships, but, other than elevations, there is no mathematical definition of the third dimension. The GSDM accommodates three-dimensional spatial data with both mathematical and geometrical integrity. Given the digital nature of spatial data appropriately defined in a 3-D model, the ways in which spatial data can be manipulated, displayed, and printed is limited only by the imagination of the modern cartographer. Efficient methods and standard procedures for generating maps from spatial data needs more consideration. Metadata and documented algorithms become critical items accompanying any cartographic product.

Softcopy Photogrammetry
Digital photogrammetry, especially that involving the practice of using a computer for data visualization, has become quite widespread and is known as softcopy photogrammetry. With use of airborne GPS control, high-quality lens calibrations, and ray tracing being modeled in efficient computer algorithms, procedures for handling digital spatial data and generating stereo images have been automated. With appropriate interfacing, the GSDM can be implemented to support a wide range of photogrammetric mapping and remote sensing applications.

National Spatial Data Infrastructure
Creation of a National Spatial Data Infrastructure (NSDI), as defined in the Executive Directive signed by President Clinton, April 11, 1994, represents enormous progress in recognizing the importance of properly managing spatial data as a national resource. The relationship between the NSDI and a GSDM need to be specifically identified and examined in light of metadata and spatial data transfer standards.

Procedures
Specific procedures for efficient implementation of a GSDM will vary according to available resources, administrative structure, and technical expertise. The GSDM does not necessarily replace other ways of handling spatial data, but as an umbrella concept, it provides an overall framework within which existing methods and procedures can be organized to ensure compatibility of spatial data. A choice to use the GSDM is voluntary and can be made independent of other spatial data users. It is completely up to the user whether the GSDM is used only for geodetic control or whether it is implemented for all spatial data files. The only dictatorial feature of using the GSDM is that each user is responsible for the form and quality of data intended to be shared with others. That is, the originator—generator—of the data is responsible for defining the shared X/Y/Z coordinates and establishing the appropriate variance/covariance values for each point. Associated metadata are still required for auditing and quality assurance purposes but, with reliable standard deviations and covariance values stored as part of the spatial data definition, user choices are much more efficient.

Solid Geometry and Vector Algebra
Although the mathematical concepts of solid geometry and vector algebra are well-proven and universally accepted, specific applications to a well-defined three-dimensional global spatial data model offers many opportunities for developing innovative ways of manipulating and using spatial data. The broad field of 3-D visualizations and CAD 3-D modeling will gain additional "real world" significance as models are attachable to existing geodetic control.

Impact on (Benefits to) Various Disciplines
Using spatial data defined in a global context with specific quality defined may be expected to provide benefits to and have an impact on many disciplines not now known or contemplated.

COST OF IMPLEMENTING THE 3-D GLOBAL SPATIAL DATA MODEL

It is not possible, at this point, to make an accurate estimate of the cost to implement the 3-D GSDM within the Southeastern Wisconsin Planning Region. At the simplest level, there is no cost because the model is herein demonstrated to be appropriate and all the equations needed to use the GSDM are included in this document. Given its use is voluntary and that existing practices can be incorporated in the GDSM, nothing prevents its immediate implementation for those applications and for those uses deemed to be appropriate.

However, there are a number of activities which can be identified as contributing to a well-organized, effective plan for widespread implementation of the GSDM within the SEWRPC Region. Many of the activities would be conducted "in-house," while others would be contracted to appropriate Federal/State agencies and/or consultants. Gross cost estimates, which depend heavily on underlying assumptions, are:

• Geoid study to calibrate GEOID96 to existing SEWRPC leveling			$ 50,000
• Development of software tailored to SEWRPC data processing procedures			50,000
• Training for SEWRPC and other spatial data technicians and professionals, 50 people @ $500 each			25,000
• Preparation of a Comprehensive Implementation Plan, including analysis of which data and activities should be 3-D based			50,000
• Data conversion of existing information:			125,000
	— NGS and WisDOT primary survey control	$ 5,000	
	— SEWRPC control survey data	20,000	
	— Existing digital spatial data files	100,000	
• Indirect costs @ 50 percent			150,000
Total			$450,000

Even if cost estimates are wrong by a factor of 2, the total investment is still far less than the Commission 1990 estimate of $7.5 million to convert existing NAD27 horizontal control survey data to the NAD83 and provides immeasurably more benefits through use of the GSDM.

BENEFITS

There are many benefits associated with adoption of the Global Spatial Data Model and it is impossible to assign realistic dollar values to them. However, the following are suggested for consideration:

- The GSDM is universally applicable worldwide in the same way it is within the SEWRPC Region.
- Responsibility for defining a relationship between an existing horizontal and vertical datum and the GSDM is local with each user and not borne by others. SEWRPC has already done that with publication of SEWRPC Technical Reports Numbers 34 and 35.
- The GSDM inherently supports definition of data quality which provides numerous administrative benefits while documenting, preserving, and promoting the integrity and value of the existing data base.
- The reputation of SEWRPC as a world leader in managing and handling spatial data is enhanced.

CONCLUSION

This report describes issues relating to use of digital spatial data in the context of modern data collection systems, storage of digital spatial data in electronic files, manipulation of spatial data, and use of spatial data. The unifying concept of the report is use of a comprehensive three-dimensional spatial data model which includes all aspects of geometry (the functional model) and data quality (the stochastic model). The mathematical concepts have all been proven in other applications but this report integrates them into a single system which has the potential of unifying procedures of spatial data between disciplines the world over.

The 3-D GPS test reported in Appendix D provides an example of how the GSDM can be applied at the local level. Although many other comparisons could be shown, the example in Appendix D documents favorable comparisons of distance (agreement with published distance within 1:160,000), azimuth (independent vectors agree with each other and show a small standard deviation of 2.2 arc seconds, and agreed with the published azimuth within 7.3 seconds), and elevation (absolute elevation agrees within 0.12 foot and elevation differences agree within first-order tolerances). The concept is proven valid.

The issue of geoid heights remains critical and needs additional attention before proceeding with plans for widespread implementation. It was hoped input specific to the geoid in Southeastern Wisconsin would be obtained from the National Geodetic Survey and included in this report. That input, except for the publication of GEOID96, is missing from this report. With regard to a possible geoid study, it is recommended that the Southeastern Wisconsin Regional Planning Commission coordinate with the National Geodetic Survey and the Wisconsin Department of Transportation to insure the best possible use of public resources.

Appendix A

MATHEMATICAL MODELS

Appendix A

MATHEMATICAL MODELS

A mathematical model is an abstract portrayal of the real world using representative figures and numbers. It gives relevance and meaning to the concepts of spatial data and location. A triangle, rectangle, or other geometrical figure drawn on a sheet of paper to represent a tract of land is a simple mathematical model. Numerical dimensions representing the lengths of the sides or the size of an angle may also be included and are part of the model. When the graphical elements are properly oriented and the lines shown with proportional lengths, the drawing can be said to be a map and the scale of the map is stated as a ratio of units on the map to units on the ground such as 1 inch on the map equals 100 feet on the ground, or as a unitless ratio such as 1:1200, also indicating then 1 inch on the map equals 100 feet on the ground.

<u>Measurements</u>
Numbers shown on a model may represent quantities which were measured and are therefore not exact. Such dimensions thus contain some uncertainty based upon the circumstance of the measurement. For example, a distance measured by Global Positioning System may be more accurate than the same distance measured by electronic distances measurement, which may be more accurate than the same distance measured by steel tape. Similarly, other measured quantities such as angles may also contain error.

<u>Errorless Dimensions</u>
Other dimensions shown on a model may be design dimensions and may be considered errorless until some attempt is made to create the physical object. The width of a five-foot sidewalk shown on a set of plans is without error until the forms are built and the concrete is poured. After the concrete sets, then the width of the sidewalk can be measured and reported. Other quantities which might be considered errorless—although not, in fact, errorless—are the published location of higher order control survey monuments upon which a current lower order survey is to be based.

<u>Derived Quantities</u>
A mathematical model may also depict derived quantities such as area, or indirect measurements which were computed from other known or measured quantities. The accuracy of such derived quantities is dependent upon the accuracy of the underlying measurements and the appropriateness of the model. Error propagation is the mechanism by which the uncertainty of computed quantities is determined.

When formally defined, every mathematical model used in surveying has two components—functional and stochastic. In practice, the functional model has been used extensively while use of the stochastic model is becoming more commonplace.

- Systematic error corrections are generally related to a functional model which represents the geometry or physical relationship between the abstraction and that being represented. A topographic map is a three-dimensional model which portrays both the planimetric location of features such as roads and buildings, while the third dimension is depicted by contour lines of constant elevation. Other examples of functional models include defining horizontal distance as the right triangle component of a slope distance or using the coefficient of thermal expansion of the steel tape when computing taping corrections.

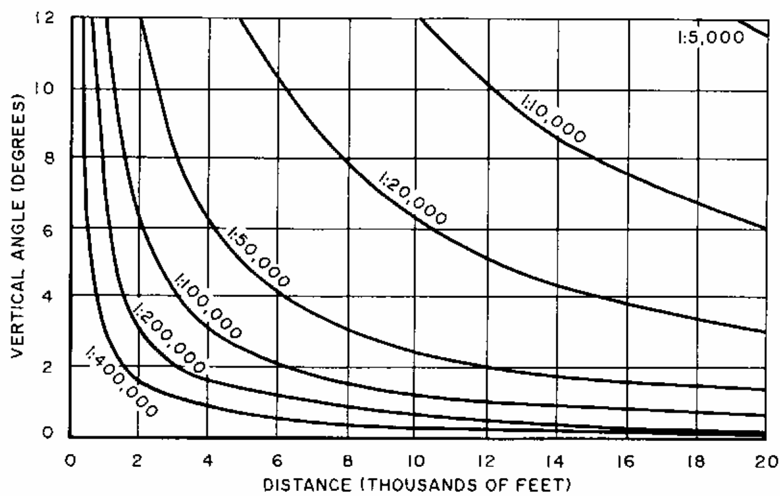
- The stochastic model accommodates random errors and represents the probabilistic characteristics of various elements of the functional model. Whether a quantity is fixed by statute, held to a previous survey, controlled by higher order instrumentation (calibration), determined by repeated measurements, or computed from a combination of known elements, the stochastic model represents the "totality of the assumptions on the statistical properties of the variables involved," (Mikhail, 1976). The standard deviation of any quantity is a statistical measure of its quality. A distance with a small standard deviation is known quite precisely while a distance with a large standard deviation is known less precisely. Use of the stochastic model is governed by rules of variance/covariance propagation.

Some functional models are more appropriate than others. To the extent one is willing to assume a flat earth, a plane triangle can be used to represent a triangular-shaped tract on the earth's surface. However, as the size of the tract increases and/or as the level of required precision is increased, the plane triangle is no longer an appropriate mathematical model, but a spherical triangle must be used. Similarly, sometimes a horizontal distance is computed assuming the plumb lines at two ends of a slope distance are parallel when in fact, they are not. If the slope distance is over 10,000 feet, the vertical angle is over 2°; and if a systematic model distortion of 1:100,000 cannot be tolerated, a more refined (complex) horizontal distance model must be used (see Figure A-1, Burkholder 1991).

The choice of a mathematical model is driven by simplicity and integrity. A simple model is generally preferred to a complex one. In the examples just cited, a flat-earth model is simple and enjoys computational integrity so long as the precision of a measurement is significantly less than a systematic error distortion imposed by the choice of a model. However, as the accuracy of modern measurement technology has increased and as the scope of application broadens, the selection of an appropriate functional model deserves careful attention. And, especially with regard to use of spatial data, the contribution of the stochastic model (positional tolerance) should be a part of that consideration.

Figure A-1

ACCURACY OF HORIZONTAL COMPONENT OF SLOPE DISTANCE IF PLUMB LINES AT ENDPOINTS ARE ASSUMED PARALLEL



Appendix B

ALGORITHMS FOR THE DIGITAL GLOBAL SPATIAL DATA MODEL

B-1: Functional Model

B-2: Stochastic Model

Appendix B-1

FUNCTIONAL MODEL

The following symbols are defined and used as:

X/Y/Z	=	Geocentric right-handed rectangular coordinates
ΔX/ΔY/ΔZ	=	Geocentric coordinate differences
e/n/u	=	Local right-handed rectangular coordinates
Δe/Δn/Δu	=	Local coordinate differences
φ/λ/h	=	Geodetic latitude/longitude (east) and ellipsoid height
a & b	=	Semi-major & semi-minor axes of reference ellipsoid
f	=	Flattening of reference ellipsoid
e^2	=	Eccentricity squared of reference ellipsoid; $e^2 = 2f - f^2$
N	=	Length of ellipsoid normal, also used for geoid height
r	=	Spatial distance from origin to point X/Y/Z
P	=	Projection of r to equatorial plane
a' b' h' φ'	=	Intermediate computational values used by Vincenty
T & U	=	Intermediate computational values used by Vincenty
S	=	Spatial slope distance between standpoint & forepoint
α	=	Geodetic azimuth at standpoint to forepoint
z or V	=	Zenith direction or vertical angle to forepoint
H	=	Orthometric height (elevation)
ΔN/Δh/ΔH	=	Changes in geoid, ellipsoid, and orthometric heights
c+r	=	Combined correction for curvature and refraction
HD(1)	=	Ground level horizontal distance (see Burkholder, 1991)

Note: (1) All distances are expressed in units of meters.
(2) Where two points are concerned, the standpoint is indicated by the subscript 1, while the forepoint is indicated by the subscript 2.

The following equations are keyed to the circled letters shown on Figure 2 in the body of this report:

A. Forward and Inverse Computations using geocentric coordinates:

<u>Forward</u>	<u>Inverse</u>	
$X_2 = X_1 + \Delta X$	$\Delta X = X_2 - X_1$	(1) & (2)
$Y_2 = Y_1 + \Delta_Y$	$\Delta_Y = Y_2 - Y_1$	(3) & (4)
$Z_2 = Z_1 + \Delta Z$	$\Delta Z = Z_2 - Z_1$	(5) & (6)

B-1. Convert geodetic latitude/longitude/ellipsoid height to geocentric X/Y/Z:

$$N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \tag{7}$$

$$X = (N + h) \cos\phi \cos\lambda \tag{8}$$

$$Y = (N + h) \cos\phi \sin\lambda \tag{9}$$

$$Z = (N [1 - e^2] + h) \sin\phi \tag{10}$$

B-2. Convert geocentric X/Y/Z to geodetic latitude/longitude/ellipsoid height:

It is difficult to invert the equations given in B-1 to obtain a closed form solution. A very good closed form approximation—which, however, breaks down for very large values of the ellipsoidal height—is given on page 232 by Hofman-Wellenhof, 1992. A common solution is to iterate equations (12) and (13) for an "exact" solution (see page 225, Leick, 1995). The recommended procedure is to assume N = 0 for the first iteration and to stop the iteration when the value of h no longer changes by a significant amount.

$$\lambda = \tan^{-1}\left(\frac{Y}{X}\right) \tag{11}$$

$$\phi = \tan^{-1}\left[\frac{Z}{\sqrt{X^2 + Y^2}}\left(1 + \frac{e^2 N \sin\phi}{Z}\right)\right] \quad \text{where} \quad N = \frac{a}{\sqrt{1 - e^2 \sin^2 \phi}} \tag{12}$$

$$h = \frac{\sqrt{X^2 + Y^2}}{\cos\phi} - N \tag{13}$$

Another option is to use a precise "once through" approximation by Vincenty (1980) which is accurate globally within 0.2 millimeter; as:

$$b = a(1 - f) \tag{14}$$

$$p^2 = X^2 + Y^2, \qquad r^2 = P^2 + Z^2 \tag*{(15) \& (16)}$$

$$h' = r - a + \frac{(a-b)Z^2}{r^2} \tag{17}$$

$$a' = a + h', \qquad b' = b + h' \qquad (18)\ \&\ (19)$$

$$\tan\phi' = \left(\frac{a'}{b'}\right)^2 \left(\frac{Z}{P}\right) \left[1 + \frac{1}{4}\,\frac{e^4 h' a (Z^2 - P^2)}{a'^4}\right] \tag{20}$$

$$\cos^2\phi' = \frac{1}{1 + \tan^2\phi'}, \qquad \sin\phi' = \cos\phi' \tan\phi' \qquad (21)\ \&\ (22)$$

$$T = \frac{(P - h'\cos\phi')^2}{a^2}, \qquad U = \frac{(Z - h'\sin\phi')^2}{b^2} \qquad (23)\ \&\ (24)$$

$$h = h' + \frac{1}{2}\left[\frac{T + U - 1}{\frac{T}{a} + \frac{U}{b}}\right] \tag{25}$$

$$\phi = \tan^{-1}\left[\left(\frac{a}{b}\right)^2 \frac{(Z - e^2 h \sin\phi')}{P}\right] \tag{26}$$

$$\lambda = \tan^{-1}\left(\frac{Y}{X}\right) \quad \text{; } \lambda \text{ is positive for east and negative for west values of longitude} \tag{27}$$

C. The conversion between Geocentric Coordinate Differences and Local Geodetic Horizon Coordinate Differences can be accomplished very efficiently with a rotation matrix or the conversions can also be done using individual equations for each component. Both methods are presented.

C-1. Geocentric Coordinate Differences can be converted to Local Coordinate Differences using the matrix form of equation (28) (Leick, 1995, Equations 7.9 and 7.10) or individually by component using equations (29), (30), and (31).

$$\begin{bmatrix}\Delta e\\ \Delta n\\ \Delta u\end{bmatrix} = \begin{bmatrix}-\sin\lambda & \cos\lambda & 0\\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi\\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi\end{bmatrix}\begin{bmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{bmatrix} \tag{28}$$

$$\Delta e = -\Delta X \sin\lambda + \Delta Y \cos\lambda \tag{29}$$

$$\Delta n = -\Delta X \sin\phi\cos\lambda - \Delta Y \sin\phi\sin\lambda + \Delta Z\cos\phi \tag{30}$$

$$\Delta u = \Delta X \cos\phi\cos\lambda + \Delta Y\cos\phi\sin\lambda + \Delta Z\sin\phi \tag{31}$$

C-2. Local Geodetic Horizon Coordinate Differences can be converted to Geocentric Coordinate Differences in similar fashion using either the matrix form in equation (32) or individually by component using equations (33), (34), and (35). See Burkholder (1993).

$$\begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} = \begin{bmatrix}-\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda\\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda\\ 0 & \cos\phi & \sin\phi\end{bmatrix}\begin{bmatrix}\Delta e\\ \Delta n\\ \Delta u\end{bmatrix} \tag{32}$$

$$\Delta X = -\Delta e\sin\lambda - \Delta n\sin\phi\cos\lambda + \Delta u\cos\phi\cos\lambda \tag{33}$$

$$\Delta Y = \Delta e\cos\lambda - \Delta n\sin\phi\sin\lambda + \Delta u\cos\phi\sin\lambda \tag{34}$$

$$\Delta Z = \Delta n\cos\phi + \Delta u\sin\phi \tag{35}$$

D. Local Geodetic Horizon Coordinate Differences are computed from terrestrial observations with equations (36, (37), and (38) (corrected as necessary for instrument calibration, atmospheric conditions, polar motion and local deflection-of-the-vertical. See Burkholder (1993)).

$$\Delta e = S\sin z\sin\alpha = HD(1)\sin\alpha \tag{36}$$

$$\Delta n = S\sin z\cos\alpha = HD(1)\cos\alpha \tag{37}$$

$$\Delta u = S\cos z \tag{38}$$

E. Equations (1) through (38) follow rules of solid geometry and vector algebra and can be used to define and express spatial relationships either globally or locally without loss of geometrical integrity. However, except for possible corrections due to deflection-of-the-vertical, gravity and level surfaces are not a part of the foregoing presentation. Given the importance of determining elevations in hydraulic and other engineering applications, the orthometric height—elevation of a point—is obtained from geocentric coordinates by way of ellipsoid heights and geoid heights using equation (39).

$$H = h - N \tag{39}$$

At the risk of oversimplification, equation (39) is very useful for computing elevations, but it presumes accurate geoid heights are known. In reality, a better method is to use a known elevation at Point 1 along with observed ellipsoid height difference from GPS measurements and modeled geoid height difference from a model such as GEOID96. In that case, the elevation of Point 2 is:

$$H_2 = H_1 + \Delta H = H_1 + \Delta h - \Delta N \tag{40}$$

$$H_2 = H_1 + (h_2 - h_1) - (N_2 - N_1) \tag{41}$$

Equations (40) and (41) are equivalent and very useful, but still limited by the accuracy of available information as, for example, about the geoid. The prudent user understands that in all cases, the value of the least accurate of the three elements in equation (39) should be computed from the other two more reliable elements. The trend being driven by current technology and ongoing research is to compute orthometric heights from ellipsoid heights and geoid heights.

In cases where the forepoint elevation is to be computed from the "up" component of the local geodetic horizon coordinates, the curvature and refraction correction can be used locally to approximate H_2 as:

$$H_2 = H_1 + \Delta H = H_1 + \Delta u + (c + r) \tag{42}$$

$$H_2 = H_1 + \Delta u + 0.0675 \frac{(\Delta e^2 + \Delta n^2)}{1{,}000{,}000} \tag{43}$$

; see equation 5.7 in Davis, Foot, Anderson, Mikhail, 1981

Figure B-1

GEOCENTRIC X, Y, Z AND GEODETIC Φ, λ, h COORDINATES

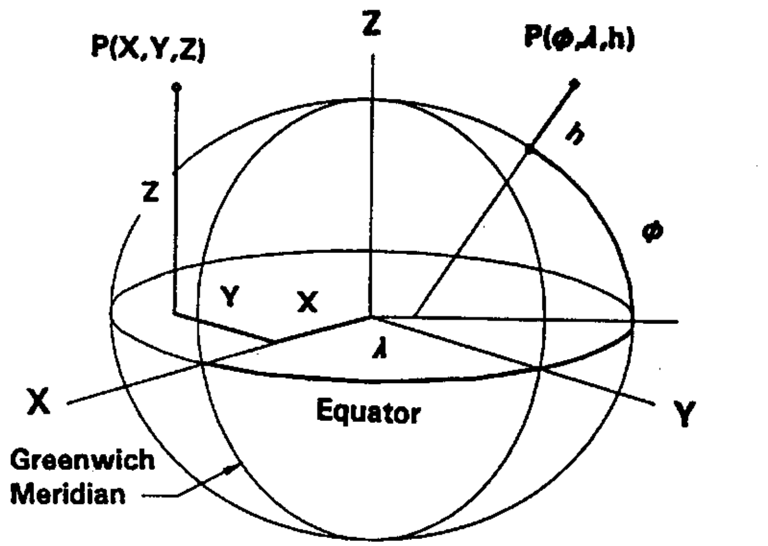
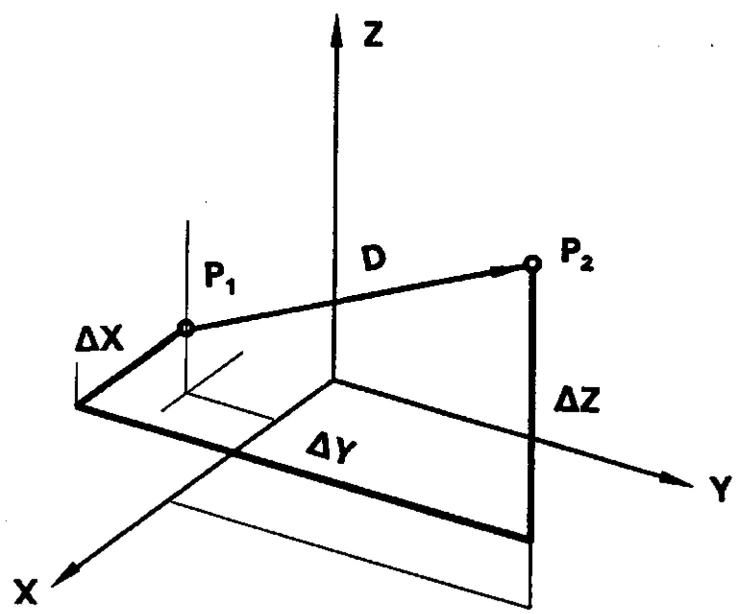


Figure B-2

GPS RELATIVE POSITIONING PROVIDES PRECISE $\Delta X/\Delta Y/\Delta Z$



$$X_2 = X_1 + \Delta X \qquad \Delta X = X_2 - X_1$$
$$Y_2 = Y_1 + \Delta Y \qquad \Delta Y = Y_2 - Y_1$$
$$Z_2 = Z_1 + \Delta Z \qquad \Delta Z = Z_2 - Z_1$$

Appendix B-2

STOCHASTIC MODEL

I. The equations listed in this section represent an application of the laws of variance/covariance propagation as described in Chapter 4 of Mikhail (1976) and make extensive use of the following matrix formulation applied to equations of the functional model:

$$\Sigma_{YY} = J_{YX} \Sigma_{XX} J^{t}_{YX} \tag{44}$$

where:

Σ_{YY} = Covariance matrix of computed result.
Σ_{XX} = Covariance matrix of variables used in computation.
J_{YX} = Jacobian matrix of partial derivatives of the result with respect to the variables.

In particular, the following symbols are in addition to those used for the functional model listed in Appendix B-1:

$\sigma_X^2, \sigma_Y^2, \sigma_Z^2$ = Variances of geocentric coordinates for a point.

$\sigma_{XY}, \sigma_{XZ}, \sigma_{YZ}$ = Covariances of geocentric coordinates for a point.

$\sigma_e^2, \sigma_n^2, \sigma_u^2$ = Variances of a point in the local reference frame.

$\sigma_{en}, \sigma_{eu}, \sigma_{nu}$ = Covariances of a point in the local reference frame.

$\sigma_{\Delta X}^2, \sigma_{\Delta Y}^2, \sigma_{\Delta Z}^2$ = Variances of geocentric coordinate differences.

$\sigma_{\Delta X\Delta Y}, \sigma_{\Delta X\Delta Z}, \sigma_{\Delta Y\Delta Z}$ = Covariances of geocentric coordinate differences.

$\sigma_{\Delta e}^2, \sigma_{\Delta n}^2, \sigma_{\Delta u}^2$ = Variances of coordinate differences in local frame.

$\sigma_{\Delta e\Delta n}, \sigma_{\Delta e\Delta u}, \sigma_{\Delta n\Delta u}$ = Covariances of coordinate differences in local frame.

$\sigma_S^2, \sigma_\alpha^2$ = Variances of local horizontal distance and azimuth.

$\sigma_{S\alpha}$ = Covariance of local horizontal distance with azimuth.

σ_z^2 = Variance of zenith direction.

II. The stochastic information for each point is stored as its geocentric covariance matrix.

A. The covariance matrix is symmetric 3 X 3. Six numbers are required to store upper (or lower) triangular values.

B. Units in the covariance matrix is meters squared.

C. Standard deviation is square root of diagonal elements.

$$\begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix}$$

III. Functional model computations supported by the stochastic model:

A. Geocentric coordinate differences from geocentric coordinates:

Matrix formulation of the functional model equations is:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \underbrace{\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}}_{J} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \tag{45}$$

The Jacobian matrix noted above is used with the general matrix variance/covariance propagation formulation, equation (44) as:

$$\Sigma_\Delta = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \sigma^2_{X_1} & \sigma_{X_1Y_1} & \sigma_{X_1Z_1} \\ \sigma_{X_1Y_1} & \sigma^2_{Y_1} & \sigma_{Y_1Z_1} \\ \sigma_{X_1Z_1} & \sigma_{Y_1Z_1} & \sigma^2_{Z_1} \end{bmatrix} & \begin{bmatrix} \sigma_{X_1X_2} & \sigma_{X_1Y_2} & \sigma_{X_1Z_2} \\ \sigma_{Y_1X_2} & \sigma_{Y_1Y_2} & \sigma_{Y_1Z_2} \\ \sigma_{Z_1X_2} & \sigma_{Z_1Y_2} & \sigma_{Z_1Z_2} \end{bmatrix} \\ \begin{bmatrix} \sigma_{X_1X_2} & \sigma_{Y_1X_2} & \sigma_{Z_1X_2} \\ \sigma_{X_1Y_2} & \sigma_{Y_1Y_2} & \sigma_{Z_1Y_2} \\ \sigma_{X_1Z_2} & \sigma_{Y_1Z_2} & \sigma_{Z_1Z_2} \end{bmatrix} & \begin{bmatrix} \sigma^2_{X_2} & \sigma_{X_2Y_2} & \sigma_{X_2Z_2} \\ \sigma_{Y_2X_2} & \sigma^2_{Y_2} & \sigma_{Y_2Z_2} \\ \sigma_{Z_2X_2} & \sigma_{Z_2Y_2} & \sigma^2_{Z_2} \end{bmatrix} \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{46}$$

Assuming no correlation between points and omitting many computational details, the result is:

$$\Sigma_\Delta = \begin{bmatrix} \sigma^2_{\Delta X} & \sigma_{\Delta X \Delta Y} & \sigma_{\Delta Y \Delta Z} \\ \sigma_{\Delta X \Delta Y} & \sigma^2_{\Delta Y} & \sigma_{\Delta Y \Delta Z} \\ \sigma_{\Delta X \Delta Z} & \sigma_{\Delta Y \Delta Z} & \sigma^2_{\Delta Z} \end{bmatrix} = \begin{bmatrix} (\sigma^2_{X_1} + \sigma^2_{X_2}) & (\sigma_{X_1Y_1} + \sigma_{X_2Y_2}) & (\sigma_{X_1Z_1} + \sigma_{X_2Z_2}) \\ (\sigma_{X_1Y_1} + \sigma_{X_2Y_2}) & (\sigma^2_{Y_1} + \sigma^2_{Y_2}) & (\sigma_{Y_1Z_1} + \sigma_{Y_2Z_2}) \\ (\sigma_{X_1Z_1} + \sigma_{X_2Z_2}) & (\sigma_{Y_1Z_1} + \sigma_{Y_2Z_2}) & (\sigma^2_{Z_1} + \sigma^2_{Z_2}) \end{bmatrix} \tag{47}$$

B. Local coordinate differences from geocentric coordinate differences:

The matrix formulation of the functional model equations is:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \underbrace{\begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}}_{J} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{48}$$

The Jacobian matrix noted above is used with the general error propagation formulation to get the covariance matrix of local coordinate differences as:

$$\Sigma_\Delta = \begin{bmatrix} \sigma^2_{\Delta e} & \sigma_{\Delta e \Delta n} & \sigma_{\Delta e \Delta u} \\ \sigma_{\Delta e \Delta n} & \sigma^2_{\Delta n} & \sigma_{\Delta n \Delta u} \\ \sigma_{\Delta e \Delta u} & \sigma_{\Delta n \Delta u} & \sigma^2_{\Delta u} \end{bmatrix} = J \begin{bmatrix} \sigma^2_{\Delta X} & \sigma_{\Delta X \Delta Y} & \sigma_{\Delta X \Delta Z} \\ \sigma_{\Delta X \Delta Y} & \sigma^2_{\Delta Y} & \sigma_{\Delta Y \Delta Z} \\ \sigma_{\Delta X \Delta Z} & \sigma_{\Delta Y \Delta Z} & \sigma^2_{\Delta Z} \end{bmatrix} J^t \tag{49}$$

C. Geocentric coordinate differences from local coordinate differences:

The matrix formulation of the functional model equations is:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \underbrace{\begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix}}_{J} \begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} \tag{50}$$

The Jacobian matrix noted above is used with the general error propagation formulation to get the covariance matrix of geocentric coordinate differences as:

$$\Sigma_\Delta = \begin{bmatrix} \sigma^2_{\Delta X} & \sigma_{\Delta X \Delta Y} & \sigma_{\Delta X \Delta Z} \\ \sigma_{\Delta X \Delta Y} & \sigma^2_{\Delta Y} & \sigma_{\Delta Y \Delta Z} \\ \sigma_{\Delta X \Delta Z} & \sigma_{\Delta Y \Delta Z} & \sigma^2_{\Delta Z} \end{bmatrix} = J \begin{bmatrix} \sigma^2_{\Delta e} & \sigma_{\Delta e \Delta n} & \sigma_{\Delta e \Delta u} \\ \sigma_{\Delta e \Delta n} & \sigma^2_{\Delta n} & \sigma_{\Delta n \Delta u} \\ \sigma_{\Delta e \Delta u} & \sigma_{\Delta n \Delta u} & \sigma^2_{\Delta u} \end{bmatrix} J^t \tag{51}$$

D. A rigorous transformation from one 3-dimensional rectangular coordinate system to another is given by a seven-parameter transformation. In matrix form, the functional model equation is:

$$\mathbf{X}_2 = s\,\mathbf{R}\,\mathbf{X}_1 + \mathbf{K} \qquad \text{where} \tag{52}$$

$\mathbf{X}_2$ = Vector of frame 2 coordinates
s = Scaler, usually assigned as 1.0
R = Rotation matrix, frame 1 to frame 2
$\mathbf{X}_1$ = Vector of frame 1 coordinates
K = Translation vector

Applying covariance propagation to that system of equations gives:

$$\Sigma_{YY} = \mathbf{J}\,\Sigma_{XX}\,\mathbf{J}^t \qquad \text{where} \tag{53}$$

Σ_{YY} = Covariance matrix of frame 2 coordinates
Σ_{XX} = Covariance matrix of frame 1 coordinates
$\mathbf{J}$ = Partial derivative matrix of frame 2 coordinates with respect to frame 1. (Rotation matrix, $\mathbf{R}_1$ or $\mathbf{R}_2$, see below)

Therefore, the covariance matrix of a point position in the local reference frame is obtained from the covariance matrix of the same point in the geocentric reference frame as:

$$\Sigma_{enu} = \begin{bmatrix} \sigma_e^2 & \sigma_{en} & \sigma_{eu} \\ \sigma_{en} & \sigma_n^2 & \sigma_{nu} \\ \sigma_{eu} & \sigma_{nu} & \sigma_u^2 \end{bmatrix} = R_1 \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix} R_1^t \quad \text{where} \tag{54}$$

$$R_1 = \begin{bmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi \end{bmatrix}$$

And, the covariance matrix of a point position in the geocentric reference frame is obtained from the covariance matrix of the same point in the local reference frame as:

$$\Sigma_{XYZ} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{YZ} & \sigma_Z^2 \end{bmatrix} = R_2 \begin{bmatrix} \sigma_e^2 & \sigma_{en} & \sigma_{eu} \\ \sigma_{en} & \sigma_n^2 & \sigma_{\nu} \\ \sigma_{eu} & \sigma_{\nu} & \sigma_u^2 \end{bmatrix} R_2^t \quad \text{where} \tag{55}$$

$$R_2 = \begin{bmatrix} -\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix}$$

R_1 and R_2 define the rotation matrix.

E. Inverse distance and azimuth from local coordinate differences:

The functional model equations for distance and azimuth are:

$$S = \sqrt{\Delta e^2 + \Delta n^2} \tag{56}$$

$$\alpha = \tan^{-1}\left(\frac{\Delta e}{\Delta n}\right) \tag{57}$$

The Jacobian matrix of partial derivatives is:

$$J = \begin{bmatrix} \frac{\partial S}{\partial \Delta e} & \frac{\partial S}{\partial \Delta n} & \frac{\partial S}{\partial \Delta u} \\ \frac{\partial \alpha}{\partial \Delta e} & \frac{\partial \alpha}{\partial \Delta n} & \frac{\partial \alpha}{\partial \Delta u} \end{bmatrix} = \begin{bmatrix} \frac{\Delta e}{S} & \frac{\Delta n}{S} & 0 \\ \frac{\Delta n}{S^2} & \frac{\Delta e}{S^2} & 0 \end{bmatrix} \tag{58}$$

Using the covariance propagation formulation, the results are:

$$\Sigma_{INV} = \begin{bmatrix} \sigma_S^2 & \sigma_{S\alpha} \\ \sigma_{S\alpha} & \sigma_\alpha^2 \end{bmatrix} = J \begin{bmatrix} \sigma_{\Delta e}^2 & \sigma_{\Delta e \Delta n} & \sigma_{\Delta e \Delta u} \\ \sigma_{\Delta e \Delta n} & \sigma_{\Delta n}^2 & \sigma_{\Delta n \Delta u} \\ \sigma_{\Delta e \Delta u} & \sigma_{\Delta n \Delta u} & \sigma_{\Delta u}^2 \end{bmatrix} J^t \tag{59}$$

F. For a new point based upon a traverse computation during which the geocentric coordinates of the point are determined, the functional model in matrix form is:

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{J} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{60}$$

The covariance matrix of the variables (which assumes no correlation between the coordinates of Point 1 and the geocentric coordinate differences) is:

$$\Sigma_{variables} = \begin{bmatrix} \begin{bmatrix} \sigma^2_{X_1} & \sigma_{X_1Y_1} & \sigma_{X_1Z_1} \\ \sigma_{X_1Y_1} & \sigma^2_{Y_1} & \sigma_{Y_1Z_1} \\ \sigma_{X_1Z_1} & \sigma_{Y_1Z_1} & \sigma^2_{Z_1} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \sigma^2_{\Delta X} & \sigma_{\Delta X\Delta Y} & \sigma_{\Delta X\Delta Z} \\ \sigma_{\Delta X\Delta Y} & \sigma^2_{\Delta Y} & \sigma_{\Delta Y\Delta Z} \\ \sigma_{\Delta X\Delta Z} & \sigma_{\Delta Y\Delta Z} & \sigma^2_{\Delta Z} \end{bmatrix} \end{bmatrix} \tag{61}$$

Applying covariance propagation, equation (44), the covariance matrix of a newly established point is:

$$\Sigma_{XYZ_2} = J\,\Sigma_{variables}\,J^t = \Sigma_{XYZ_1} + \Sigma_{\Delta X\Delta Y\Delta Z} \tag{62}$$

Note that the covariance matrix for Point 1 is either presumed known or zero. The remaining portion of this section is addressed to obtaining the covariance matrix of the geocentric coordinate differences. Two identifiable options are:

1. Using GPS processing results, the variance/covariance values of the geocentric coordinate differences for a base line are available and used.

2. The geocentric coordinate differences of the vector from Point 1 to Point 2 is obtained by rotating local coordinate differences to geocentric coordinate differences using equation (32). The covariance matrix of the geocentric coordinate differences is obtained from the covariance matrix of the local coordinate differences using equation (51).

   The next question addressed is that of obtaining the covariance matrix of local coordinate differences from conventional "total station" surveying measurements. An underlying assumption[1] here (which is nearly true, but not quite) is that the azimuth of each line is an independent quantity. The functional model for local (mark to mark) coordinate differences is:

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \begin{bmatrix} S\ \sin z\ \sin\alpha \\ S\ \sin z\ \cos\alpha \\ S\ \cos z \end{bmatrix} \quad \text{where} \tag{63}$$

S = Slope distance, standpoint to forepoint.
α = Azimuth, standpoint to forepoint.
z = Zenith direction, standpoint to forepoint. (If reciprocal zenith directions are used, the "curvature" portion of the correction should be removed.)

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[1]*The same assumption has been widely adopted whenever the Compass Rule is used to adjust a traverse whose azimuths were determined with a transit or theodolite instead of a compass or gyroscope. Although formulation of the equations gets tedious and more storage is required for larger matrices, the stochastic model will competently handle correlation between points and correlation from one course to another.*

The Jacobian matrix is obtained as the matrix of partial derivatives with respect to the observed quantities as:

$$J = \begin{bmatrix} \frac{\partial \Delta e}{\partial S} & \frac{\partial \Delta e}{\partial z} & \frac{\partial \Delta e}{\partial \alpha} \\ \frac{\partial \Delta n}{\partial S} & \frac{\partial \Delta n}{\partial z} & \frac{\partial \Delta n}{\partial \alpha} \\ \frac{\partial \Delta u}{\partial S} & \frac{\partial \Delta u}{\partial z} & \frac{\partial \Delta u}{\partial \alpha} \end{bmatrix} = \begin{bmatrix} \sin z \sin\alpha & S\cos z \sin\alpha & S\sin z\cos\alpha \\ \sin z\cos\alpha & S\cos z\cos\alpha & -S\sin z\sin\alpha \\ \cos z & -S\sin z & 0 \end{bmatrix} \tag{64}$$

The variance/covariance matrix of the observed quantities (variables) is a diagonal matrix of variances due to independence of the measurements. This assumption is related to, but not the same as the earlier assumption of independence of azimuth from course to course. Using the Jacobian matrix and the variance matrix of observations in equation (44), the covariance matrix of local coordinate differences is:

$$\Sigma_{\Delta e \Delta n \Delta u} = \begin{bmatrix} \sigma^2_{\Delta e} & \sigma_{\Delta e \Delta n} & \sigma_{\Delta e \Delta u} \\ \sigma_{\Delta e \Delta n} & \sigma^2_{\Delta n} & \sigma_{\Delta n \Delta u} \\ \sigma_{\Delta e \Delta u} & \sigma_{\Delta n \Delta u} & \sigma^2_{\Delta u} \end{bmatrix} = J \begin{bmatrix} \sigma^2_S & 0 & 0 \\ 0 & \sigma^2_z & 0 \\ 0 & 0 & \sigma^2_\alpha \end{bmatrix} J^t \tag{65}$$

IV. Options for using the stochastic model are:

A. When the stochastic model is not used for a point:

1. No standard deviation or covariance values are input.

2. The covariance matrix is set to zero.

3. The geocentric X/Y/Z position is used as being errorless.

B. When the geocentric covariance matrix for a point is defined at the same time as its X/Y/Z coordinates:

1. Input as standard deviations of X/Y/Z:

   a. Units of meters input (stored as variance - meters squared)
   b. No correlation data input. Off diagonal elements are zero.
   c. Different components may have different values.

2. Full covariance matrix is input for each point:

   a. Units of meters squared
   b. Six elements input and stored
   c. Caution - care is required to assure input of covariance values which are mathematically consistent. If inappropriate values of covariance are input, negative variances (incorrect) in another reference frame may be the result.

C. When the geocentric covariance matrix for a point is computed from the local covariance matrix which is determined at the same time the point is defined. Input options for local covariance matrix are:

1. Input as standard deviations of e/n/u:

   a. Units of meters input (stored as variance - meters squared)
   b. No correlation data input. Off diagonal elements are zero.
   c. Different components may have different values.

2. Full local covariance matrix is input for each point:

   a. Units of meters squared
   b. Six elements input and stored
   c. Caution - care is required to assure input of covariance values which are mathematically consistent. If inappropriate values of covariance are input, negative variances (incorrect) in another reference frame may be the result.

3. The geocentric covariance matrix for the point defined is computed using equation (55).

D. When Point 2 is established by adding user supplied geocentric coordinate differences to the geocentric coordinates of Point 1. The covariance matrix of Point 2 is found using equation (62). Options for obtaining the covariance matrix of the geocentric coordinate differences include:

1. No covariance data are available.

   a. No covariance values are input.
   b. Covariance matrix of geocentric coordinate differences is set to zero.
   c. The uncertainty of Point 2 is the same as at Point 1.

2. Input standard deviations of geocentric coordinate differences.

   a. Units of meters input (stored as variance - meters squared)
   b. No correlation data input. Off diagonal elements are zero.
   c. Different components may have different values.

3. Full covariance matrix is input for geocentric coordinate differences of the vector:

   a. Units of meters squared
   b. Six elements input and stored
   c. Caution - care is required to assure input of covariance values which are mathematically consistent. If inappropriate values of covariance are input, negative variances (incorrect) in another reference frame may be the result.

E. When Point 2 is established by adding geocentric coordinate differences, obtained by rotating local coordinate differences to the geocentric reference frame, to the geocentric coordinates of Point 1. In this case, equation (51) is used before the covariance matrix of Point 2 can be found using equation (62). Options for obtaining the covariance matrix of the local coordinate differences include:

1. No covariance data are available.

   a. No covariance values are input.
   b. Covariance matrix of local coordinate differences is set to zero.
   c. The uncertainty of Point 2 is the same as at Point 1.

2. Input standard deviations of local coordinate differences.

   a. Units of meters input (stored as variance - meters squared)
   b. No correlation data input. Off diagonal elements are zero.
   c. Different components may have different values.

3. Full covariance matrix is input for geocentric coordinate differences of the vector:

   a. Units of meters squared
   b. Six elements input and stored
   c. Caution - care is required to assure input of covariance values which are mathematically consistent. If inappropriate values of covariance are input, negative variances (incorrect) in another reference frame may be the result.

4. Use equation (65) to obtain the covariance matrix of the local coordinate differences based upon independent measurements of slope distance, zenith directions, and azimuth.

   a. Only standard deviations are input. Units are:
      i) Meters for slope distance.
      ii) Radians for angular values. Programs can be written to accept other units of input.
   b. No correlation data input. Off diagonal elements are zero.
   c. Each independent observation has its own standard deviation.
   d. If vertical angles are used, two options are:
      i) Change equations (63) and (64).
      ii) Compute zenith direction from vertical angle. (Standard deviation is same for vertical or zenith.)

Appendix C

GEOID 96 MODEL

Appendix C

GEOID 96 MODEL

README file 9-oct-96 dgm/das

The GEOID96 GEOID MODELS

You have received these models on CD-ROM, or downloaded them from the National Geodetic Survey (NGS) web site, the NGS FTP site, the NGS bulletin board system, or have received the models on individual floppy disks.

Among the files you have received are:

GEOID.EXE the geoid interpolation program (GEOID.FOR is source code) (version 3.0)

DOSXMSF.EXE 32-bit DOS extender (needed for GEOID.EXE)

AREA.PAR text file of the filenames of geoid height grids

GE096NE.GEO the GEOID96 grid for the Northeastern U.S. 36-50N, 89- 66W
GE096NC.GEO the GEOID96 grid for the Northcentral U.S. 36-50N, 107- 84W
GE096NW.GEO the GEOID96 grid for the Northwestern U.S. 36-50N, 125-102W
GE096SE.GEO the GEOID96 grid for the Southeastern U.S. 24-38N, 89- 66W
GE096SC.GEO the GEOID96 grid for the Southcentral U.S. 24-38N, 107- 84W
GE096SW.GEO the GEOID96 grid for the Southwestern U.S. 24-38N, 125-102W
GE096AN.GEO the GEOID96 grid for North Alaska 60-72N, 179-128W
GE096AS.GEO the GEOID96 grid for South Alaska 51-63N, 179-128W
GE096HW.GEO the GEOID96 grid for the Principal Hawaiian Islands
GE096PR.GEO the GEOID96 grid for Puerto Rico - Virgin Islands

GEOGRD.EX utility program for sub-area extraction and format conversion (GEOGRD.FOR is the source code)

To Install (after uncompressing the files)

1) Make a subdirectory on your hard disk (example: mkdir c:\geoid96).
2) Copy the various geoid files into that subdirectory.
   copy *.* c:\geoid96 /v (for example)
3) Repeat step 2) as required for your other sets of geoid files.
   (If you have also received G96SSS model files, do not place them in the same subdirectory as your GEOID96 files.)
   (if you have installed from floppies, the files are put in the designated subdirectory.)

4) Check your AUTOEXEC.BAT and CONFIG.SYS files to insure compliance with the following notes:

Note 1: DOSXMSF.EXE must either be present in the same directory as GEOID.EXE, or, it must be in a directory in your DOS PATH environment variable. (such as: c:\dos, for example) DOSXMSF.EXE may be freely reproduced and distributed, without royalty.

Note 2: You must have a statement FILES=25 (or a number greater than 25) in your CONFIG.SYS file.

To Execute
Type GEOID , and follow the prompts.

To Terminate

You can stop the program at any time by the Control C key combination.

BUT, PLEASE DON'T START YET. PLEASE KEEP READING THIS DOCUMENT.

How Program GEOID Works

The various geoid height grids are stored in the ".GEO" files. Program GEOID will assume that the files in your local directory with a .GEO extension are geoid height files. You can operate with as few as one GEO file, or as many as 15. When the program interpolates a given point, it checks an internal list of .GEO boundaries, and uses the earliest list entry whose boundaries contain that point. The order in which the .GEO file names appear on the opening screen indicates the order in which the .GEO files are searched.

The AREA.PAR File

AREA.PAR is a plain, ASCII text file. It specifies the order in which .GEO files are to be used. If you have a favorite .GEO file, put it at the top of the AREA.PAR list. There is no problem in having overlapping .GEO files, nor is there any problem in having nested .GEO files. The AREA.PAR file specifies which geoid files are available and their priority of use.

PLEASE NOTE:

The AREA.PAR file we distribute contains the names of all the GEOID96 grid files. You may not have received them all; you may not want them all. This is not a problem. If a .GEO file name is in the AREA.PAR file, but not in the local directory, then a warning message is issued, and program GEOID proceeds with the files that are available. You must have an entry in AREA.PAR for each .GEO file to be searched.

An Example:
You just wish to work with the GEOID96 - Northwest file. So, load AREA.PAR into your favorite line editor, and delete the lines referring to the other geoid regions. You may now delete those .GEO fites without receiving the warning messages on the opening screen of program GEOID. Save the updated AREA.PAR as plain ASCII text.

Data Input

You can key data by hand, point by point, or you can create an input file using a text editor. Several file formats are provided, including the NGS “Blue Book” format. These formats are detailed in a “Help” menu option which appears if you specify an input file name. That file doesn't need to exist if you are only going to look at the supported formats in the “Help”.

Data Output

Results are collected into an output file. The default name of these files is GEOID.OUT, but you can use any legal file name you choose. (A word of advice: Don't use misleading extensions such as .EXE, .GEO, .BAT, etc.) The format of the output file is linked to the format of the input file to maintain consistency.

The GEOID96 Model

The GEOID96 model was computed on October 1, 1996 using over 1.8 million terrestrial and marine gravity values. The method of computation uses a Fast Fourier Transform (FFT) technique to compute the detailed geoid structure, which is then combined with an underlying EGM96 geopotential model. The result is a gravimetric geoid height grid with a 2' X 2' spacing in latitude and longitude (2' x 4' in Alaska), referred to the Geodetic Reference System 1980 (GRS 80) normal ellipsoid in an International Terrestrial Reference System 1994 (ITRF94) frame. Then, by means of NAD83 GPS ellipsoidal heights on NAVD88 benchmark data, plus known relationships between NAD83 and the ITRF94 reference frames, a conversion is applied to generate the final GEOID96 geoid model. This conversion causes the GEOID96 model to be biased relative to a geocentric ellipsoid; but, this bias is deliberate. The GEOID96 model was developed to support direct conversion between NAD83 GPS ellipsoidal heights and NAVD88 orthometric heights.

When comparing the GEOID96 model with GPS ellipsoidal heights in the NAD83 reference frame and leveling in the NAVD88 datum, it is seen that GEOID96 has roughly a 3-cm accuracy (one sigma) in the regions of GPS benchmark coverage. In those states with sparse (150km+) GPS benchmark Coverage, less point accuracy may be evident; but relative accuracy at about a 1 to 2 part-per-million level, or better, should still be obtained. For users with less stringent accuracy requirements, simple height conversions with GEOID96 in the conterminous United States can be sufficient. For users with more stringent accuracy requiements, please see the section entitled “Deriving Orthometric Heights From GPS,” later in this document. Users should be aware that GPS ellipsoid height error, by itself, can be significantly greater than error in geoid height differences.

States with Sparse GPS Benchmark Coverage

As of the date of computation of GEOID96, the states with sparse GPS benchmark coverage are: Arkansas, Illinois, Indiana, Iowa, Kansas, Minnesota, Missouri, North Dakota, South Dakota, and West Virginia. This does not mean that the GEOID96 model can not be used in these states. It does mean that users may not see the same absolute accuracy when compared to other parts of the United States with denser GPS benchmark coverage. As stated above, relative accuracy may reach 1-2 PPM. Even so, the major components of the datum relationships between NAD83 and NAVD88 in all of the lower 48 states have been incorporated into the GEOID96 geoid model. As a rule, one can expect better results with GEOID96, relative to GEOID93, in any part of the United States.

Alaska, Hawaii, Puerto Rico and the Virgin islands

It must be emphasized that the GEOID96 models in Alaska, Hawaii, Puerto Rico, and the Virgin Islands were NOT, repeat, NOT computed by incorporating a conversion surface based on GPS benchmarks. This was due to a shortage of reliable NAD83 GPS ellipsoidal heights on NAVD88 benchmarks in these regions. The GEOID96 geoid models provided in these areas are relative to a geocentric, GRS80 ellipsoid as were earlier GEOID93 and GEOID90 models. For this reason, users should refer to the section entitled “Deriving Orthometric Heights From GPS,” later in this document.

Due to poorer data coverage, error estimates for GEOID96 in these regions are larger. Long-wavelength errors may be as large as 4-5 parts-per-million in some areas. Particular care must be used in computing heights in the tectonically active areas in southern Alaska. Crustal motion may exceed 1 meter even after accounting for the shift of the 1964 Prince William Sound Earthquake.

The National Imagery and Mapping Agency

The National Imagery and Mapping Agency (NIMA), which incorporates the former Defense Mapping Agency (DMA), has been of immense help in this endeavor. NIMA has provided a major portion of the NGS land gravity data set. NIMA has also been instrumental in the creation of the various 30" and 3" elevation grids in existence. And, NIMA was a partner in the joint project to compute the new global geopotential model, EGM96. Although the work of the NIMA generally precludes public recognition, their cooperation is gratefully acknowledged.

GSFC/NIMA Geopotential Model, EGM96

The Goddard Space Flight Center (GSFC) and the National Imagery and Mapping Agency (NIMA) have been engaged in a joint project to compute an improved global spherical harmonic model of the Earth's geopotential. This model incorporates the latest satellite tracking data, as well as altimeter data from TOPEX/Poseidon, ERS-1, and the Geosat Geodetic Mission. EGM96 also incorporates new surface and marine gravity data covering the globe, including the former Soviet Union.

EGM96 is a global geopotential model expressed as spherical harmonic coefficients complete to degree and order 360. Therefore, the shortest wavelength this model can exhibit is one degree, and its resolution is one-half degree (about 50 km). Although this model does not reproduce geoid structure at very fine resolution, it is global. We thank the many members of the project team for making this model available.

Deriving Orthometric Heights From GPS

One key problem is deciding which orthometric height datum to use. NGVD29 is not a sea-level datum, and the heights are not true orthometric heights. The datum of NAVD88 is selected to maintain reasonable conformance with existing height datums, and its Helmert heights are good approximations of true orthometric heights. And, while differential ellipsoidal heights obtained from GPS are precise, they are often expressed in the NAD83 datum, which is not exactly geocentric. In addition, GEOID96 rests upon an underlying EGM96 global geopotential model, and EGM96 does possess some error of commission.

This leads to a warning:

Do not expect the difference of a GPS ellipsoidal height at a point and the associated GEOID96 height to exactly match the vertical datum you need. The results will be close when converting NAD83 GPS ellipsoidal heights into NAVD88 elevations but, maybe not accurate enough for your requirement.

However, one can combine the precision of differential carrier phase GPS with the precision of GEOID96 height differences, to approach that of leveling.

Include at least one existing benchmark in your GPS survey (preferably many benchmarks). The difference between the published elevation(s) and the height obtained from differencing your adopted GPS ellipsoidal height and the GEOID96 model, could be considered a “local orthometric height datum correction.” If you are surveying an extensive area (100+ km), and you occupy a lot of benchmarks, then you might detect a trend in the corrections up to a one part-per-million level. This may be error in the GEOID96 model.

We do not currently consider geoid-corrected GPS orthometric heights as a substitute for geodetic leveling in meeting the Federal Geodetic Control Subcommittee(FGCS) standards for vertical control networks. Studies are underway, and many less stringent requirements can be satisfied by geoid modeling. Widespread success has been achieved with the preceeding models, GEOID93 and GEOID90.

The GEOGRD Utility Program

GEOGRD -- This converts to and from “.GEO” binary files and ASCII text files. It can also be used to extract subgrids in the process of conversion. For example: one can make a .GEO grid for the state of Colorado by using GE096NW.GEO, “converting” from binary, .GEO into binary, .GEO, and specifying the Colorado state boundaries.

A Technical Note on Program GEOID

Some users prefer to write their own interpolation software. If you do, please be aware that there is a loss of precision in the grid file headers for grid spacings of 2' (or 4'). This is accomodated in program GEOID 3.00 by internally recomputing the grid spacing in subroutine GRIDS. You might need to place similar code in your interpolation software, depending upon how it was written.

-- (Example Fortran 77 code) --

```
***   patch for inexact headers (due to 2' spacing)
      idxl=idnint(DX1*3600.d0)
      DX(NAREA) = dble(idxl)/3600.d0

      idyl=idnint(DY1*3600.d0)
      DY(NAREA) = dble(idyl)/3600.d0

***   DX(NAREA) = DXl          old code
***   DY(NAREA) = DYI          old code
```

Future Plans

A research effort is underway to improve geoid height estimates in the future, perhaps at the 1-cm accuracy level. One important direction is integrating gravity data with GPS and geodetic leveling measurements, and the study of error in GPS ellipsoid heights and in the NAVD88 vertical datum. It is likely that this research, in conjunction with the completion of the state upgrade GPS surveys, will yield a significant improvement to our geoid model in 1999.

For More Information

For Products Available From the National Geodetic Survey:

Information Services Branch
National Geodetic Survey, NOAA, N/NGS12
1315 East-West Highway, SSMC3, Station 9202
Silver Spring, MD 20910-3282
301-713-3242 fax: 301-713-4172

For Information on GEOID96 and Future Research:

Dr. Dennis G. Milbert
National Geodetic Survey, NOAA, N/NGS5
1315 East-West Highway, SSMC3, Station 9349
Silver Spring, MD 20910-3282
301-713-3202
Internet: dennis@ngs.noaa.gov

Dr. Dru A. Smith
National Geodetic Survey, NOAA, N/NGS5
1315 East-West Highway, SSMC3, Station 9316
Silver Spring, MD 20910-3282
301-713-3202
Internet: dru@ngs.noaa.gov

Visit our web site:
http://www.ngs.noaa.gov/GEOID/geoid.html

Best Wishes!

README file 9-oct-96 dgm/das

Map C-1

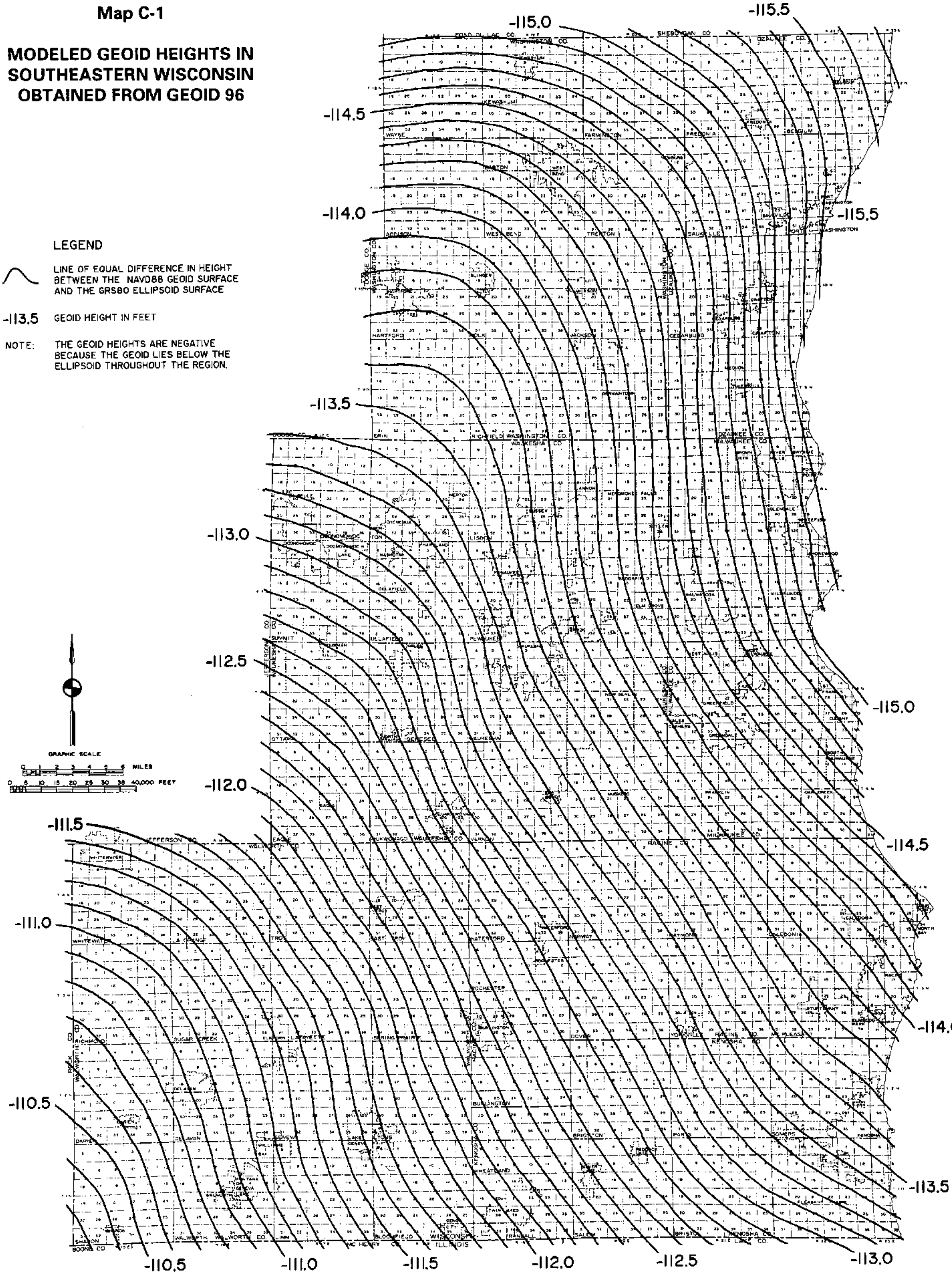
MODELED GEOID HEIGHTS IN SOUTHEASTERN WISCONSIN OBTAINED FROM GEOID 96

LEGEND

LINE OF EQUAL DIFFERENCE IN HEIGHT BETWEEN THE NAVD88 GEOID SURFACE AND THE GRS80 ELLIPSOID SURFACE

-113.5 GEOID HEIGHT IN FEET

NOTE: THE GEOID HEIGHTS ARE NEGATIVE BECAUSE THE GEOID LIES BELOW THE ELLIPSOID THROUGHOUT THE REGION.



Appendix D

3-D GPS TEST EXAMPLE

Appendix D

TEST OF 3-D MODEL WITH HARN-BASED GPS DATA

GPS Data by: Aero-Metric, Inc.
Sheboygan, Wisconsin 53082

Computations by: Earl F. Burkholder, PS, PE
Consulting Geodetic Engineer
Circleville, Ohio 43113

'BURKORD(TM)' COMPUTES 3-D COORDINATE GEOMETRY POSITIONS FOR SPATIAL DATA UTILIZING GPS VECTORS, LOCAL COORDINATE DIFFERENCES AND 3-D SURVEYING MEASUREMENTS.

COPYRIGHT (C) 1996 AND
ALL RIGHTS RESERVED BY:
GLOBAL COGO, INC
P.O. BOX 13240
CIRCLEVILLE, OHIO 43113

USE OF BURKORD(TM) LICENSED TO:
Earl F.Burkholder
Global COGO, Inc.
P.O. Box 13240
Circleville, Ohio 43113

USER: EARL F. BURKHOLDER
DATE: JANUARY 4, 1997

PROGRAM: BURKORD(TM) - VERSION 8A, DECEMBER 1996 S/N 8AC96000
DATA FILE: SEWRPC-4.DAT
OUTPUT FILE: SEWRPC-4.TST

CLIENT/AGENCY: SOUTHEASTERN WISCONSIN REGIONAL PLANNING COMMISSION
JOB/PROJECT: TEST OF GPS DATA IN 3-D GLOBAL SPATIAL DATA MODEL

```
(A) DEFINE 4410 43 25 17.242370   -88 8 4.573890   234.2970 M   WEST BEND GPS
     COVAR MATRIX  E/N/U/EN/EU/NU .00E+00 .00E+00 .00E+00 .00E+00 .00E+00 .00E+00 METERS SQD

(B) DEFINE X/Y/Z  4412  160207.6560 M  -4666182.2810 M   4331031.0070 M  MILWAUKEE GPS
     COVAR MATRIX  X/Y/Z/XY/XZ/YZ .00E+00 .00E+00 .00E+00 .00E+00 .00E+00 .00E+00 METERS SQD

    A LISTING OF POINTS IN ACTIVE PROJECT IS:

(C) 4410  151041.3221 -4637606.0218 4361788.8258 .000000 .000000 .000000 .000000 .000000 .000000 WEST BEND GPS
    4412  160207.6560 -4666182.2810 4331031.0070 .000000 .000000 .000000 .000000 .000000 .000000 MILWAUKEE GPS

(D) FORWARD BY 3-D  DX/DY/DZ  4410 TO 35   9384.6631  -11599.0587  -12636.7703  NE    8-9-21
     COVAR MATRIX  DX/DY/DZ/DXY/DXZ/DYZ .36E-04 .36E-04 .36E-04 .00E+00 .00E+00 .00E+00   METERS SQD

(E) FORWARD BY 3-D  DX/DY/DZ 4410 TO 36    8581.9504  -11635.6817  -12647.9758  N 1/4  8-9-21
     COVAR MATRIX  DX/DY/DZ/DXY/DXZ/DYZ .36E-04 .36E-04 . 36E-04 .00E+00 .00E+00 .00E+00  METERS SQD

(F) FORWARD BY 3-D  DX/DY/DZ 4412 TO 1035   218.3461   16977.2109   18121.0554  NE    8-9-21
     COVAR MATRIX  DX/DY/DZ/DXY/DXZ/DYZ .64E-04 .64E-04 .64E-04 .00E+00 .00E+00 .00E+00   METERS SQD

(G) FORWARD BY 3-D  DX/DY/DZ 4412 TO 1036  -584.3658   16940.5886   18109.8493  N 1/4  8-9-21
     COVAR MATRIX  DX/DY/DZ/DXY/DXZ/DYZ .64E-04 .64E-04 .64E-04 .00E+00 .00E+00 .00E+00   METERS SQD

A LISTING OF POINTS IN ACTIVE PROJECT IS:

     4410  151041.3221 -4637606.0218 4361788.8258 .000000 .000000 .000000 .000000 .000000 .000000 WEST BEND GPS
     4412  160207.6560 -4666182.2810 4331031.0070 .000000 .000000 .000000 .000000 .000000 .000000 MILWAUKEE GPS
(H)    35  160425.9852 -4649205.0805 4349152.0555 .000036 .000036 .000036 .000000 .000000 .000000 NE 8-9-21
       36  159623.2725 -4649241.7035 4349140.8500 .000036 .000036 .000036 .000000 .000000 .000000 N 1/4 8-9-21
     1035  160426.0021 -4649205.0701 4349152.0624 .000064 .000064 .000064 .000000 .000000 .000000 NE 8-9-21
     1036  159623.2902 -4649241.6924 4349140.8563 .000064 .000064 .000064 .000000 .000000 .000000 N 1/4 8-9-21
```

RE-ORDER SEQUENCE OF POINTS IN DATA FILE

AN EXPANDED LISTING OF POINTS 1 TO 5000

```
    35 NE  8-9-21                                         X          Y          Z               E          N          U
LAT (N+S-)   43 15 54.648270  X:   160425.9852   X   .36E-04                             E   .36E-04
LON (E+W-)  -88  1 25.426572  Y: -4649205.0805   Y   .00E+00   .36E-04                   N   .00E+00   .36E-04
EL HGT            220.3092 M  Z:  4349152.0555   Z   .00E+00   .00E+00   .36E-04         U   .00E+00   .00E+00   .36E-04

    36 N 1/4  8-9-21                                      X          Y          Z               E          N          U
LAT (N+S-)   43 15 54.184254  X:   159623.2725   X   .36E-04                             E   .36E-04
LON (E+W-)  -88  2  1.052986  Y: -4649241.7035   Y   .00E+00   .36E-04                   N   .00E+00   .36E-04
EL HGT            219.1746 M  Z:  4349140.8500   Z   .00E+00   .00E+00   .36E-04         U   .00E+00   .00E+00   .36E-04

    1035  NE   8-9-21                                     X          Y          Z               E          N          U
    LAT (N+S-)   43 15 54.648651  X:   160426.0021   X   .64E-04                             E   .64E-04
(I) LON (E+W-)  -88  1 25.425807  Y: -4649205.0701   Y   .00E+00   .64E-04                   N   .00E+00   .64E-04
    EL HGT            220.3068 M  Z:  4349152.0624   Z   .00E+00   .00E+00   .64E-04         U   .00E+00   .00E+00   .64E-04

   1036 N 1/4   8-9-21                                    X          Y          Z               E          N          U
LAT (N+S-)   43 15 54.184635  X:   159623.2902   X   .64E-04                             E   .64E-04
LON (E+W-)  -88  2  1.052185  Y: -4649241.6924   Y   .00E+00   .64E-04                   N   .00E+00   .64E-04
EL HGT            219.1713 M  Z:  4349140.8563   Z   .00E+00   .00E+00   .64E-04         U   .00E+00   .00E+00   .64E-04

    4410  WEST BEND GPS                                   X          Y          Z               E          N          U
LAT (N+S-)   43 25 17.242372  X:   151041.3221   X   .00E+00                             E   .00E+00
LON (E+W-)  -88  8  4.573891  Y: -4637606.0218   Y   .00E+00              .00E+00        N   .00E+00   .00E+00
EL HGT            234.2970 M  Z:  4361788.8258   Z   .00E+00   .00E+00   .00E+00         U   .00E+00   .00E+00   .00E+00

    4412  MILWAUKEE GPS                                   X          Y          Z               E          N          U
LAT (N+S-)   43  2 30.415115  X:   160207.6560   X   .00E+00                             E   .00E+00
LON (E+W-)  -88  2   .931147  Y: -4666182.2810   Y   .00E+00   .00E+00                   N   .00E+00   .00E+00
EL HGT            198.7958 M  Z:  4331031.0070   Z   .00E+00   .00E+00   .00E+00         U   .00E+00   .00E+00   .00E+00
```

INVERSE BETWEEN POINTS

```
    35 NE   8-9-21
X =   160425.9852  LAT (N+S-)   43 15 54.648270  +/-  .0060 METERS   N
Y = -4649205.0805  LON (E+W-)  -88  1 25.426572  +/-  .0060 METERS   E        STANDARD DEVIATIONS
Z =  4349152.0555  EL HGT           220.3092 M   +/-  .0060 METERS   U

     DELTA X/Y/Z WITH SIGMAS  -802.7127M +/-  .008M   -36.6230M +/-  .008M  -11.2055M +/-  .008M
(J)  DELTA E/N/U WITH SIGMAS  -803.4982M +/-  .008M   -14.2727M +/-  .008M   -1.1851M +/-  .008M
     LOCAL PLANE INV: DIST      803.6250M +/-  .008M  N AZI. = 268 58 56.45 +/-  2.2 SEC

    36 N 1/4  8-9-21
X =   159623.2725  LAT (N+S-)   43 15 54.184254  +/-  .0060 METERS   N
Y = -4649241.7035  LON (E+W-)  -88  2  1.052986  +/-  .0060 METERS   E        STANDARD DEVIATIONS
Z =  4349140.8500  EL HGT           219.1746M    +/-  .0060 METERS   U

     DELTA X/Y/Z WITH SIGMAS   802.7127M +/-  .008M    36.6230M +/-  .008M   11.2055M +/-  .008M
(K)  DELTA E/N/U WITH SIGMAS   803.4967M +/-  .008M    14.3679M +/-  .008M    1.0840M +/-  .008M
     LOCAL PLANE INV: DIST     803.6251M +/-  .008M  N AZI. = 88 58 32.04 +/-  2.2 SEC

    35 NE   8-9-21
X =   160425.9852  LAT (N+S-)   43 15 54.648270  +/-  .0060 METERS   N
Y = -4649205.0805  LON (E+W-)  -88  1 25.426572  +/-  .0060 METERS   E        STANDARD DEVIATIONS
Z =  4349152.0555  EL HGT           220.3092 M   +/-  .0060 METERS   U

     DELTA X/Y/Z WITH SIGMAS     .0169M +/-  .010M      .0104M +/-  .010M     .0069M +/-  .010M
     DELTA E/N/U WITH SIGMAS     .0172M +/-  .010M      .0117M +/-  .010M    -.0024M +/-  .010M
     LOCAL PLANE INV: DIST       .0209M +/-  .010M  N AZI. = 55 44 22.55 +/-****** SEC

   1035  NE 8-9-21
X =   160426.0021  LAT (N+S-)   43 15 54.648651  +/-  .0080 METERS   N
Y = -4649205.0701  LON (E+W-)  -88  1 25.425807  +/-  .0080 METERS   E        STANDARD DEVIATIONS
Z =  4349152.0624  EL HGT           220.3068 M   +/-  .0080 METERS   U

     DELTA X/Y/Z WITH SIGMAS  -802.7119M +/-  .011M   -36.6223M +/-  .011M  -11.2061M +/-  011M
(L)  DELTA E/N/U WITH SIGMAS  -803.4974M +/-  .011M   -14.2727M +/-  .011M   -1.1860M +/-  .011M
     LOCAL PLANE INV: DIST     803.6241M +/-  .011M  N AZI. = 268 58 56.46 +/-  2.9 SEC
```

1036 N 1/4 8-9-21
X = 159623.2902 LAT (N+S-) 43 15 54.184635 +/- .0080 METERS N
Y = -4649241.6924 LON (E+W-) -88 2 1.052185 +/- .0080 METERS E STANDARD DEVIATIONS
Z = 4349140.8563 EL HGT 219.1713M +/- .0080 METERS U

DELTA X/Y/Z WITH SIGMAS 802.7119M +/- .011M 36.6223M +/- .011M 11.2061M +/- .011M
(M) DELTA E/N/U WITH SIGMAS 803.4958M +/- .011M 14.3678M +/- .011M 1.0849M +/- .011M
LOCAL PLANE INV: DIST = 803.6243M +/- .011M N AZI. = 88 58 32.04 +/- 2.9 SEC

1035 NE 8-9-21
X = 160426.0021 LAT (N+S-) 43 15 54.648651 +/- .0080 METERS N
Y = -4649205.0701 LON (E+W- -88 1 25.425807 +/- .0080 METERS E STANDARD DEVIATIONS
Z = 4349152.0624 EL HGT 220.3068 M +/- .0080 METERS U

DELTA X/Y/Z WITH SIGMAS -218.3461M +/- .008M -16977.2109M +/- .008M -18121.0554M +/- .008M
DELTA E/N/U WITH SIGMAS -803.6854M +/- .008M -24819.2508M +/- .008M -69.9467M +/- .008M
LOCAL PLANE INV: DIST = 24832.2597M +/- .008M N AZI. = 181 51 16.84 +/- .1 SEC

4412 MILWAUKEE GPS
X = 160207.6560 LAT (N+S-) 43 2 30.415115 +/- .0000 METERS N
Y = -4666182.2810 LON (E+W-) -88 2 .931147 +/- .0000 METERS E STANDARD DEVIATIONS
Z = 4331031.0070 EL HGT 198.7958 M +/- .0000 METERS U

DELTA X/Y/Z WITH SIGMAS -584.3658M +/- .008M 16940.5886M +/- .008M 18109.8493M +/- .008M
DELTA E/N/U WITH SIGMAS -2.7298M +/- .008M 24805.0575M +/- .008M -27.9549M +/- .008M
LOCAL PLANE INV: DIST = 24805.0577M +/- .008M N AZI. = 359 59 37.30 +/- .1 SEC

1036 N 1/4 8-9-21
X = 159623.2902 LAT (N+S-) 43 15 54.184635 +/- .0080 METERS N
Y = -4649241.6924 LON (E+W-) -88 2 1.052185 +/- .0080 METERS E STANDARD DEVIATIONS
Z = 4349140.8563 EL HGT 219.1713 M +/- .0080 METERS U

DELTA X/Y/Z WITH SIGMAS -.0177M +/- .010M -.0111M +/- .010M -.0063M +/- .010M
DELTA E/N/U WITH SIGMA -.0181M +/- .010M -.0118M +/- .010M .0033M +/- .010M
LOCAL PLANE INV: DIST .0216M +/- .010M N AZI. = 236 54 43.71 +/-****** SEC

(N) 36 N 1/4 8-9-21
X = 159623.2725 LAT (N+S-) 43 15 54.184254 +/- .0060 METERS N
Y = -4649241.7035 LON (E+W-) -88 2 1.052986 +/- .0060 METERS E STANDARD DEVIATIONS
Z = 4349140.8500 EL HGT 219.1746 M +/- .0060 METERS U

DELTA X/Y/Z WITH SIGMAS -8581.9504M +/- .006M 11635.6817M +/- .006M 12647.9758M +/- .006M
DELTA E/N/U WITH SIGMAS -8177.6420M +/- .006M 17382.0664M +/- .006M- -13.8431M +/- .006M
LOCAL PLANE INV: DIST = 19209.6346M +/- .006M N AZI. = 334 48 16.88 +/- .1 SEC

4410 WEST BEND GPS
X = 151041.3221 LAT (N+S-) 43 25 17.242372 +/- .0000 METERS N
Y = -4637606.0218 LON (E+W-) -88 8 4.573891 +/- .0000 METERS E STANDARD DEVIATIONS
Z = 4361788.8258 EL HGT 234.2970M +/- .0000 METERS U

DELTA X/Y/Z WITH SIGMAS 9384.6631M +/- .006M -11599.0587M +/- .006M -12636.7703M +/- .006M
DELTA E/N/U WITH SIGMAS 9002.1224M +/- .006M -17356.7821M +/- .006M -43.9928M +/- .006M
LOCAL PLANE INV: DIST = 19552.3935M +/- .006M N AZI. = 152 35 11.10 +/- .1 SEC

35 NE 8-9-21
X = 160425.9852 LAT (N+S-) 43 15 54.648270 +/- .0060 METERS N
Y = -4649205.0805 LON (E+W-) -88 1 25.426572 +/- .0060 METERS E STANDARD DEVIATIONS
Z = 4349152.0555 EL HGT 220.3092 M +/- .0060 METERS U

INVERSE BETWEEN POINTS

4410 WEST BEND GPS
X = 151041.3221 LAT (N+S-) 43 25 17.242372 +/- .0000 METERS N
Y = -4637606.0218 LON (E+W- -88 8 4.573891 +/- .0000 METERS E STANDARD DEVIATIONS
Z = 4361788.8258 EL HGT 234.2970 M +/- .0000 METERS U

DELTA X/Y/Z WITH SIGMAS 9166.3339M +/- .000M -28576.2592M +/- .000M -30757.8188M +/- .000M
DELTA E/N/U WITH SIGMAS 8231.2747M +/- .000M -42176.7852M +/- .000M -180.5304M +/- .000M
LOCAL PLANE INV: DIST = 42972.4923M +/- .000M N AZI. = 168 57 24.80 +/- .0 SEC

```
4412    MILWAUKEE GPS
X =   160207.6560  LAT (N+S-)   43  2    30.415115    +/-    .0000 METERS    N
Y = -4666182.2810  LON (E+W-) -88  2      .931147    +/-    .0000 METERS    E      STANDARD DEVIATIONS
Z =  4331031.0070  EL HGT               198.7958 M   +/-    .0000 METERS    U

   DELTA X/Y/Z WITH SIGMAS   -9166.3339M +/-   .000M   28576.2592M +/-  .000M    30757.8188M +/-  .000M
   DELTA E/N/U WITH SIGMAS   -8180.3829M +/-   .000M   42186.9295M +/-  .000M     -109.5319M +/-  .000M
   LOCAL PLANE INV: DIST =   42972.7319M +/-   .000M   N AZI. = 349  1  33.88 +/-      .0 SEC

   4410   WEST BEND GPS
X =   151041.3221  LAT (N+S-)   43  25   17.242372    +/-    .0000 METERS    N
Y = -4637606.0218  LON (E+W-) -88   8     4.573891    +/-    .0000 METERS    E      STANDARD DEVIATIONS
Z =  4361788.8258  EL HGT               234.2970 M   +/-    .0000 METERS    U
```

A LISTING OF POINTS IN ACTIVE PROJECT IS:

	35	160425.9852	-4649205.0805	4349152.0555	.000036	.000036	.000036	.000000	.000000	.000000	NE 8-9-21
	36	159623.2725	-4649241.7035	4349140.8500	.000036	.000036	.000036	.000000	.000000	.000000	N 1/4 8-9-21
(O)	1035	160426.0021	-4649205.0701	4349152.0624	.000064	.000064	.000064	.000000	.000000	.000000	NE 8-9-21
	1036	159623.2902	-4649241.6924	4349140.8563	.000064	.000064	.000064	.000000	.000000	.000000	N 1/4 8-9-21
	4410	151041.3221	-4637606.0218	4361788.8258	.000000	.000000	.000000	.000000	.000000	.000000	WEST BEND GPS
	4412	160207.6560	-4666182.2810	4331031.0070	.000000	.000000	.000000	.000000	.000000	.000000	MILWAUKEE GPS

Following is a listing of data file **SEWRPC-4.DAT:**

```
SOUTHEASTERN WISCONSIN REGIONAL PLANNING COMMISSION
TEST OF GPS DATA IN 3-D GLOBAL SPATIAL DATA MODEL
     35,    160425.9852, -4649205.0805, 4349152.0555, 3.600000000E-05, 3.600000000E-05,
    3.600000000E-05, 0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 'NE  8-9-21'
     36,    159623.2725, -4649241.7035, 4349140.8500, 3.600000000E-05, 3.600000000E-05,
    3.600000000E-05, 0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 'N 1/4  8-9-21'
   1035,    160426.0021, -4649205.0701, 4349152.0624, 6.400000000E-05, 6.400000000E-05,
(P) 6.400000000E-05, 0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 'NE  8-9-21'
   1036,    159623.2902, -4649241.6924, 4349140.8563, 6.400000000E-05, 6.400000000E-05,
    6.400000000E-05, 0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 'N 1/4 8-9-21'
   4410,    151041.3221, -4637606.0218, 4361788.8258, 0.000000000E+00, 0.000000000E+00,
    0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 'WEST BEND GPS'
   4412,    160207.6560, -4666182.2810, 4331031.0070, 0.000000000E+00, 0.000000000E+00,
    0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 0.000000000E+00, 'MILWAUKEE GPS'
```

Comments on Test of 3-D Model With HARN-Based GPS Points

The test consisted of 4 points occupied simultaneously by 4 GPS receivers. Two points, "West Bend GPS" and "Milwaukee GPS," are part of the Wisconsin HARN and the 3-dimensional positions are published by the National Geodetic Survey. The other two points lie approximately midway between the HARN points and are one-half mile apart. They are U. S. Public Land Survey System corners and were previously connected to the SEWRPC horizontal and vertical control network.

The GPS data were collected on November 25, 1996, by Aero-Metric, Inc. of Sheboygan, Wisconsin. Second-order orthometric heights (NGVD29 elevations) for the two HARN stations were also determined on the same date by differential leveling from nearby benchmarks.

The following comments are keyed to the circled letters on the foregoing computer print-out.

(A) Station "West Bend GPS" is assigned point number 4410 and is defined by its latitude/longitude/height. The HARN station is assumed to be errorless and the covariance matrix is filled with zeros. Control data values are from the NGS CD-ROM data base.

(B) Station "Milwaukee GPS" is assigned point number 4412 and is defined by its geocentric X/Y/Z coordinates. Its position is also assumed to be errorless and indicated by the covariance matrix filled with zeros. Control data values are from the NGS CD-ROM data base.

(C) The listing shows the format with which points and their positions are stored in the data base. The listing shows one point per line but, as shown in item (P), the actual data file contains two lines per point.

(D) The NE corner of Section 8, Township 9 North, Range 21 East, (NE 8-9-21) is assigned point number 35 and is established from point number 4410 (West Bend GPS) using the observed GPS base line vector components and standard deviations.

(E) The North Quarter Corner of Section 8, Township 9 North, Range 21 East, (N 1/4 8-9-21) is assigned point number 36 and is established from point number 4410 (West Bend GPS) using the observed GPS base line vector components and standard deviations.

(F) The NE corner of Section 8, Township 9 North, Range 21 East, (NE 8-9-21) is assigned point number 1035 and is established from point number 4412 (Milwaukee GPS) using the observed GPS base line vector components and standard deviations.

(G) The North Quarter Corner of Section 8, Township 9 North, Range 21 East, (N 1/4 8-9-21) is assigned point number 1036 and is established from point number 4412 (Milwaukee GPS) using the observed GPS base line vector components and standard deviations.

(H) All points in the project are listed in the order they were defined/established.

(I) The data file point sequence has been reordered and the point values are printed in the "expanded" mode which shows the following data for each:
- Point number and name
- Geodetic coordinates (derived on command, not stored)
- Geocentric coordinates (stored in the data file)
- Covariance matrix in the geocentric system (stored in the data file)
- Covariance matrix in the local system (derived on command, not stored)

(J) Inverse between points 35 and 36 as established from "West Bend GPS." Features shown are:
- Geodetic and geocentric coordinates are printed for each point.
- The local direction standard deviations are printed.
- The geocentric coordinate differences are given along with their sigmas.
- The local coordinate differences (latitude & departure) are given along with their standard deviations.
- The "up" component and its standard deviation is given as the perpendicular distance from the tangent plane to the forepoint. This is close, but not the same as vertical difference. A curvature and refraction correction is needed to make it a vertical difference (see equation 43 on page 29).
- The local horizontal distance and standard deviation are listed.
- The azimuth from north and its standard deviation are given with respect to the meridian through the standpoint.

(K) Inverse between points 36 and 35 as established from "West Bend GPS." Compared to the previous inverse, the following should be noted:
- The geocentric coordinate differences are identical, except for sign.
- Local coordinate differences are slightly different, that is correct.
- The horizontal distances are slightly different. They are in slightly different tangent planes.
- As in plane surveying, the azimuth is obtained as $\tan^{-1}(\Delta e/\Delta n)$. Each azimuth is computed with respect to the meridian through the standpoint. Since meridians are not parallel, the difference between "forward" and "back" azimuth is 180° +/- the convergence of the meridians.

(L) Inverse between points 1035 and 1036 (the same two corners) as established from "Milwaukee GPS." Although observed simultaneously, this azimuth was computed independently of the inversed distance as established from "West Bend GPS." Note agreement as 803.624 meters compared to 803.625 meters. These are horizontal ground distances. Expressed in U. S. Survey Feet, the distance is 2,636.56 feet.

(M) Inverse from 1036 to 1035 as established from "Milwaukee GPS." The horizontal distance is nearly identical to the distance from 1035 to 1036 and the azimuth is from 1036 to 1035 is with respect to meridian through point number 1036.

(N) Other inverses between points.

(O) Listing of all points in the project in sequential order.

(P) Items (A) through (O) are in output file SEWRPC-4.TST generated using BURKORD™ Version 8A. This last item is a listing of the actual data file generated while using the program. The data file SEWRPC-4.DAT can be used and added to in subsequent computational sessions. Note that, after the two header lines, there are two lines required per point defined/stored.

Comparison of local horizontal distances:

The following steps were used to compare ground level horizontal distances (derived from existing state plane coordinates) with the 3-D horizontal distances:

- Using CORPSCON 4.11, the NAD27 state plane coordinates were used to determine the NAD27 latitude/longitude coordinates, the grid scale factors, and the convergence angle at points NE 8-9-21 (Point No. 36) and NI/4 8-9-21 (Point No. 35).

- The elevation reduction factor at each point is computed assuming the earth's radius is 20,906,000 feet and using the orthometric height (NGVD29) elevation for each point provided by SEWRPC.

- The combined grid reduction factor was determined for the line between the section corners as the product of the average grid scale factor and the mean elevation factor.

Determination of Combined Grid Reduction Factor

Station	NGVD29 Elevation	Elevation Factor	Grid Scale Factor	Combined Factor
NE 8-9-21 (Point No. 35)	837.537	0.99995994	0.999935354	0.99989529
(Average)	835.649	0.99996003	0.999935357	0.99989539
NI/4 8-9-21 (Point No. 36)	833.760	0.99996012	0.999935360	0.99989548

Distances Between Point No. 35 and Point No. 36

	Meters	Feet
Grid Distance	803.536	2,636.242
Combined Grid Scale Factor	0.99989539	0.99989539
Horizontal Ground Distance	803.620	2,636.517
3-D Horizontal Distance	803.625	2,636.533
Difference	0.005	0.016
Ratio of Precision	1:160,000	1:160,000

Comparison of azimuths:

The 3-D system provides the true geodetic azimuth from one point to another directly as $\tan^{-1}(\Delta e/\Delta n)$ using the local components obtained by rotating the geocentric components to local. Different answers are obtained for the "forward" and "back" azimuths because the meridians through the standpoint and the forepoint are not parallel. For the instant example, the 3-D azimuths are:

Point No. 36 to Point No. 35	AZ = 88°58' 32.0"	(as determined from West Bend GPS)
Point No. 1036 to Point No. 1035	AZ = 88°58' 32.0"	(as determined from Milwaukee GPS)
Point No. 35 to Point No. 36	AZ = 268° 58' 56.5"	(as determined from West Bend GPS)
Point No. 1035 to Point No. 1036	AZ = 268° 58' 56.5"	(as determined from Milwaukee GPS)

Azimuths obtained from state plane coordinate inverses are grid azimuths which change by exactly 180° when going opposite directions on the same line.

Convergence of the meridians must be applied at each point for which true geodetic azimuth is desired. Geodetic azimuths between Point No. 35 and Point No. 36 are obtained as:

Line	Grid Azimuth	Convergence	Geodetic Azimuth
Point No. 35 to Point No. 36	267° 37' 35.1"	01° 21' 28.7"	268° 59' 03.8"
Point No. 36 to Point No. 35	87° 37' 35.1"	01° 21' 04.2"	88° 58' 39.3"

In each case, the 3-D geodetic azimuth differs from the state plane based geodetic azimuth by 7.3 arc seconds. That difference represents an orientation accuracy of about 1/30,000 (well within the 1/10,000 needed for local control).

Comparison of elevation differences:

The following data were used to compare GPS derived elevation differences with observed differential leveling differences:

- Ellipsoid heights, h, at each of the 4 test points obtained from the geocentric X/Y/Z coordinates of same.
- The modeled elevation difference between NGVD29 and NAVD88 was obtained from VERTCON Version 2.0.
- The modeled geoid heights (N) were obtained from GEOID96.

In each case, the GPS derived elevation and elevation differences are found using the following relationships and are summarized below:

NAVD88 elevation = ellipsoid height - GEOID96 value
NGVD29 elevation = ellipsoid height - GEOID96 value - VERTCON value

Summary of Elevations and Differences

Station Name	Ellipsoid Height	Geoid 96 Value	VERTCON Value	NGVD29 by GPS	NGVD29 Published	Difference
West Bend GPS (4410	234.297 m	-34.891 m	-0.051 m	269.239 m	269.211 m	0.028 m 0.092 ft
Milwaukee GPS (4412)	198.796 m	-34.881 m	-0.084 m	233.761 m	233.742 m	0.019 m 0.062 ft
NE 8-9-21 (35)	220.308 m	-34.941 m	-0.067 m	255.316 m	255.282 m	0.034 m 0.112 ft
N 1/4 8-9-21 (36)	219.173 mi	-34.925 m	-0.067 m	254.165 m	254.131 m	0.034 m 0.112 ft

Stations From - To	Distance	GPS ΔH	Pub ΔH	Difference Meters	Difference Feet	Coefficient, Feet*√Miles
35 - 36	0.499 mi	-1.151 m	-1.151 m	0.000 m	0.000'	0.000
35 - 4410	12.15 mi	13.923 m	13.929 m	0.006 m	0.020'	0.006
35 - 4412	15.43 mi	-21.555 m	-21.540 m	-0.015 m	-0.049'	-0.013
36 - 4410	11.94 mi	15.074 m	15.080 m	0.006 m	-0.020'	0.006
36 - 4412	15.41 mi	-20.404 m	-20.389 m	-0.015 m	-0.049'	-0.013
4410 - 4412	26.70 mi	-35.478 m	-35.469 m	-0.009 m	-0.030'	-0.006

FGCS Leveling Criteria

First Order	Class I	3 mm √km or	0.012 ft√mi
	Class II	4 mm √km or	0.017 ft√mi

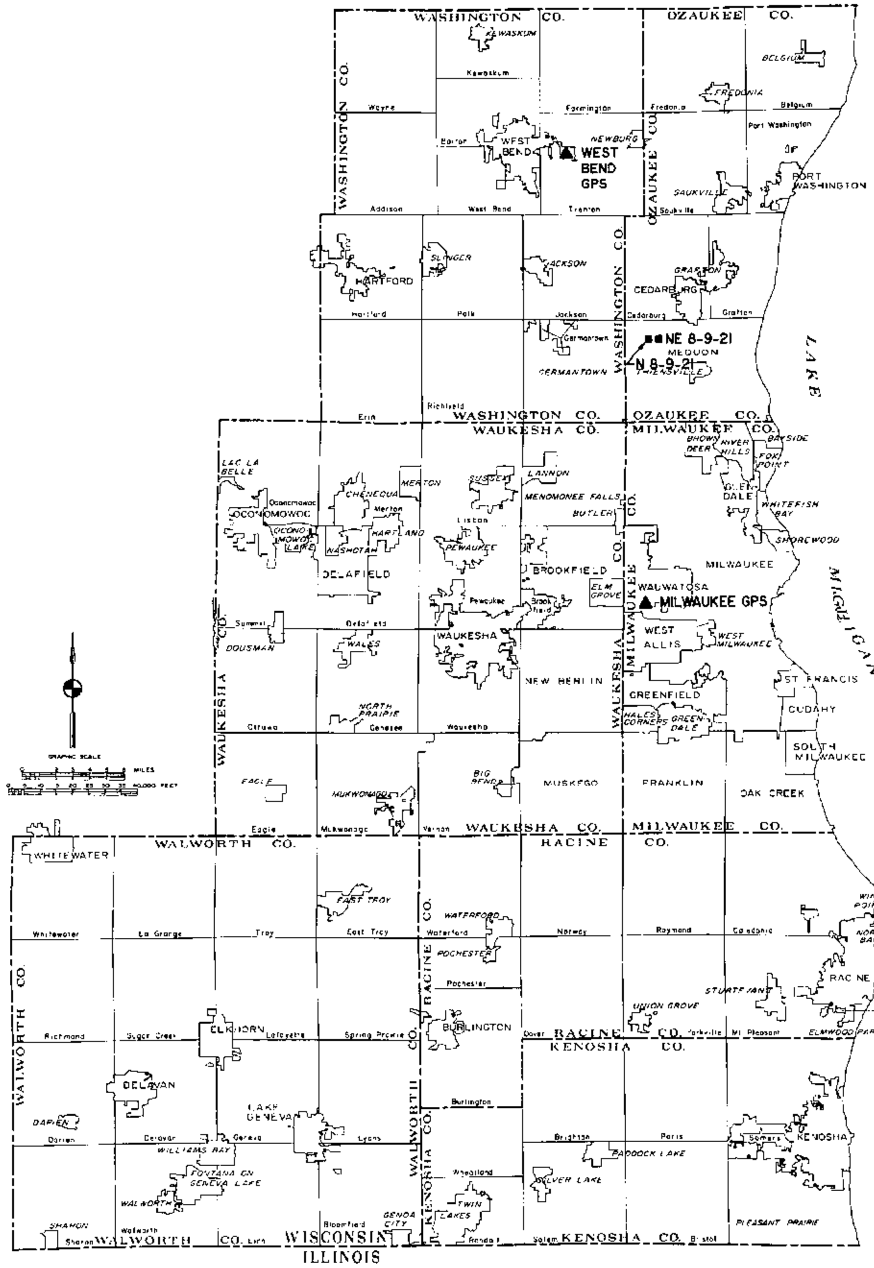
Summary:

Among others, this test shows that:

- The 3-D model provides accurate answers efficiently.
- GPS can be used to obtain high quality elevations.
- GEOID96 can be very useful in modeling geoid height differences.

Map D-1

LOCATION OF STATIONS USED FOR THE 3-D GPS TEST



Appendix E

REFERENCES

Appendix E

REFERENCES

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